THEORY OF MACHINE

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Simple mechanism

Machine

It is a device which recieve energy in some form and utilise this & perform some usefull work.

Machine dynamics

It is the study of relative motion between parts of machine considering appⁿ of force.

Mechanism

It is formed when one part of machine fixed & relative force motion of other parts w.r.t fixed part can be determined is known as mechanism.

E.g - Type writer.

→ A simple mechanism have 4 Parts
→ It is used for translating motion,
→ If a mechanism consists of morethen four parts, then it is compound mechanism.

Link

→ It is a part of machine
→ It is a resistant body (which does not change it's shape) which move related to some other part.

purpose of link

→ It act as a support
→ It guides other link.

E.g. crank.

Types of link

It is of two types
a) Rigid
b) Flexible.

Rigid link

It is a type of link which does not undergoes any deformation while transmitting motion.

e.g:- connecting rod, crank shaft

Flexible link

It is a type of link which partially deformed while transmitting motion

Eg:- Belt, rope.

Difference betn machine & structure

Machine	Structure
→ Relative motion exists betn links	→ Relative motion does not exist.
→ It runs by electric motor	→ It does not run by electric motor.
→ The parts of machine transmit only motion	→ The member of structure act as a support or, carries load only.

Constrained motion

→ The motion which takes place with in boundary is known as constrained motion.

→ It is three type i) Complited
ii) In complited
iii) Partial

i) complited constrained motion

The motion betwn a pair of element takes place in a definite direction irrespective of direction of force applied.

eg → Piston cylinder arrangement.

ii) Incomplited constriained motion

The motion betn a pair of element takes place more then one direction is known as incomplited constrained motion.

e.g:- a circular bar moving in circularhole.

iii) Partically constrained motion

The motion betn a pair of element takes place only in one direction with the help of some enternal agent (Force)

e.g.- shaft in a footstep bearing.

Scanned with CamScanner

Pair

When two parts of machine come in contact with each other, then a pair is formed.

Kinematic pair

It is a pair of element permit relative motion betn them, then it is called as kinematic pair.

Classification of kinematic pair

Kinematic pair are classified in to two types.

1) According to relative motion
2) According to their type of connection.

1) According to the relative motion

It is classified into 5 types

a) Sliding pair
b) Turning pair
c) Rolling pair
d) Spherical pair
e) Screw pair

a) Sliding pair

This type of pair is formed by two elements which are so conected that one element slide with other element.

b) Turning pair

This type of pair formed by two elements which are so conected that, one element turns w.r.t other.

e.g:- Lathe spindle support in a head stock.

d) Spherical pair

This type of pair is formed by two elements which are so connected that one spherical member turns with a fixed element.

c) Rolling pair

This type of pair is formed by two elements which are so connected that one element rolls w.r.t other

e.g:- ball bearing, Roller bearing.

e) Screw pair

This type of pair is formed by two elements which are so connected that one element turns w.r.t other by a screw thread. e.g - Nut-bolt

2) According to the type of connection

a) Higher pair
b) Lower pair
c) Open pair
d) Closed pair.

a) Higher pair

when a pair of element have point contact or line contact then this is called as higher pair

e.g:- Cam & follower.

b) Lower pair

when a pair of two elements having surface contact in motion is called as lower pair.

e.g - Piston & cylinder.

c) Open pair

When a pair of two elements which are held together mechanically but they are kept in contact with the help of an external agent.

e.g. Cam & follower.

d) Closed pair

When a pair of two elements which are held together mechanically only, then it is called as closed pair.

e.g:- Piston, cylinder

N.B

The lower pair is also called as a closed pair

Kinematic chain

→ It is the combination of kinematic pairs, which are so connected that each link forms a part of two pairs and relative motion betn them takes place in a definite ~~zone~~. direction

→ For kinematic chain

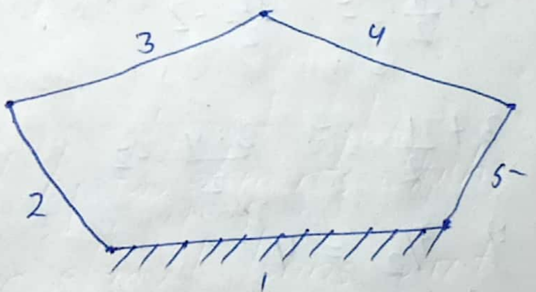
$\boxed{L = 2P - 4}$ & $\boxed{J = \frac{3L}{2} - 2}$

where l = no. of links
P = no. of pairs
J = no. of Joints

N.B

i) If ~~Eq.~~ L.H.S = R.H.S , kinematic chain is constrained
ii) If L.H.S < R.H.S, Kinematic chain is unconstrained
iii) If L.H.S > R.H.S, Kinematic chain is locked.

eg. $L = 2P - 4$
$\Rightarrow 2 \times 5 - 4$
$\Rightarrow 6$



$J = \frac{3L}{2} - 2$

$= \frac{3 \times 6}{2} - 2 = 7$

L.H.S < R.H.S, Unconstrained kinematic chain

Types of Joints

There are three types of Joints

i) Binary
ii) Trinary
iii) Quarternary.

i) <u>Binary</u>

When two links are Joined at the same point then it is called as binary Joint

According to A.W Klein, $\boxed{J + H/2 = \frac{3L}{2} - 2}$

where H = higher pair

Q) Find if the kinematic chain is constrained or unconstrained.

$J = 3$
$L = 4$
$H = 1$

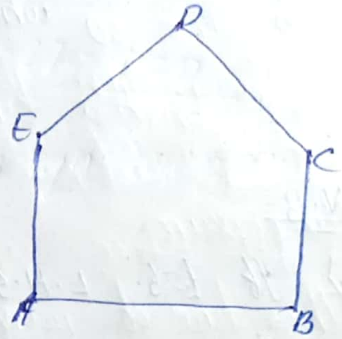
$J + \frac{H}{2} = \frac{3L}{2} - 2$

$\Rightarrow 3 + \frac{1}{2} = \frac{3 \cdot 4}{2} - 2$

$\Rightarrow 7/2 = 4$

$\Rightarrow 3.5 < 4$

Unconstrained K.C



$J = 5$
$L = 5$
$H = 0$

$J + \frac{H}{2} = \frac{3L}{2} - 2$

$\Rightarrow 5 + 0/2 = \frac{3 \times 5}{2} - 2$

$\Rightarrow 5 = 5.5$

$\Rightarrow 5 < 5.5$

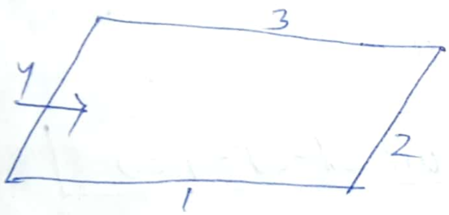
Unconstrained K.C chain.

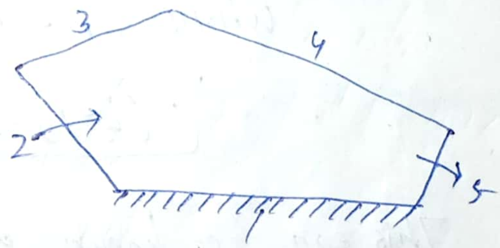
Degree of freedom

when a link in a chain is sufficient to define the relative position of other link into a definite direction, is called as degree of freedom.

Mobility or No. of degree of freedom

It is defined as no. of input parameter which must be independently controlled in order to define the relative position of other link into perticular definite direction.

① 

② 

* In fig -(1), the input parameter is given to link 4, so mobility (n) = 1

* In fig (2), the input parameter is given to link (2) & (5), so mobility (n) = 2

When no. of links are connected by no. of binary Joints (J) then mobility, $n = 3(L-1) - 2J - H$

where H = higher pair

N.B

1) When n = 0, then it is said to be locked

2) When n = 1, 2 or more, then it is called as single D.O.F or double D.O.F or, as so.

3) When n = -1, It is called over bridged structure.

→ It is applied to the mechanism with angle D.O.F (lower pairs are use) and mobility of mechanism is unity

→ According to kutzbach criterion

$$n = 3(L-1) - 2J - H$$

where , $n = 1$, $H = 0$

$$\boxed{3(L-1) - 2J \text{ or } 3l - 2J - 4 = 0}$$

This eqn is known as Grublers eqn for plain mechanism.

Inversion of mechanism

The method of obtaining different mechanism by fixing different links in a kinematic chain is known as inversion of mechanism.

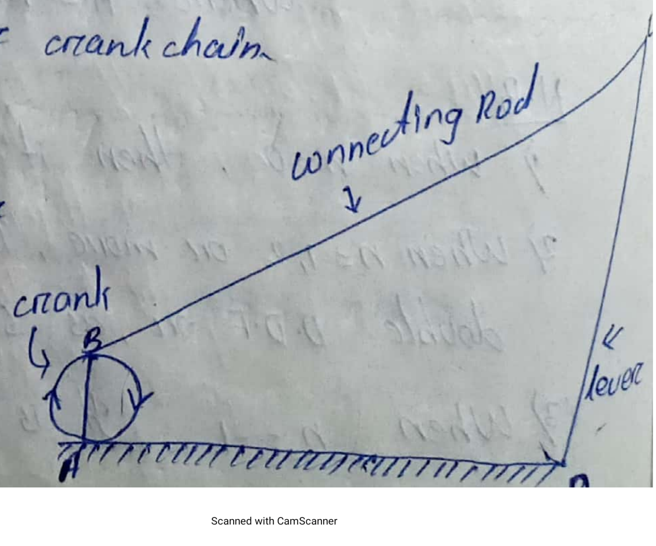
Types of kinematic chain

It is classified into following types

i) 4 bar chain
ii) Single slider crank chain
iii) Double slider crank chain

4 BAR CHAIN

In 4 bar chain there are four links. each link forms a turning pair at point A,B,C,D



→ In 4 bar chain, length of links are differend

→ In 4 bar chain, the sum of length of short and Longest shouldnot be greater then that of remaining link. This is known as Grashofs law.

∴ AB + BC   CD + AD

Description

→ In 4 bar mechanism, shortest link AB will make a complitely revltn to other link. So link AB is known as crank.

→ Link BC which is conected to crank & link CD is called conecting rod.

→ Link CD is partially rotate or, oscillate is known as lever.

→ Link AD is known as fixed link. this type of mechanism is used for transmitting rotary motion to oscillaeting motion

Inversion of 4-bar chain

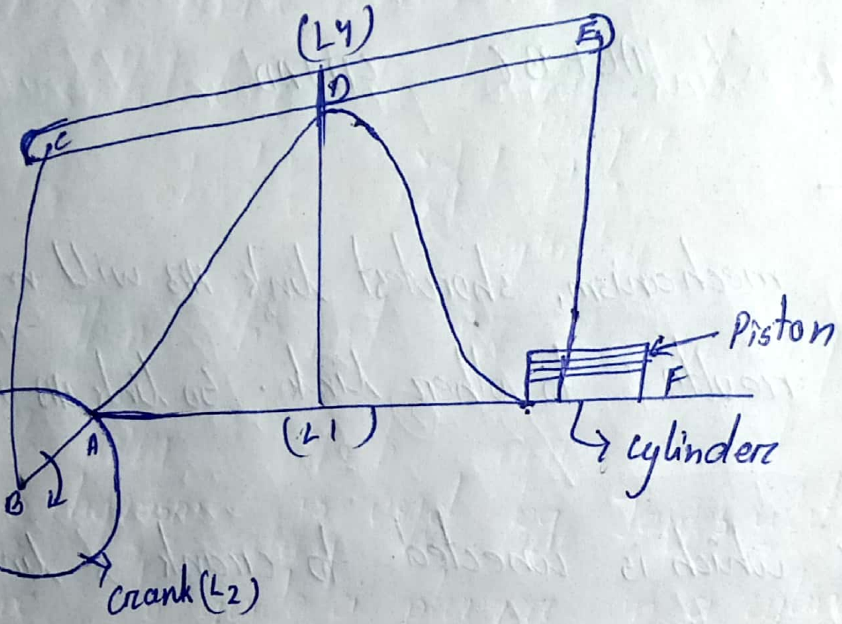
In 4-bar chain, there are 3-types of Inversion

i) Beam Engine
ii) Double crank mechanism
iii) Watt indicator mechanism or double lever.

Beam engine

In this arrangement, engine cylinder is placed on a frame or fixed beam. it consists of 4 bar.

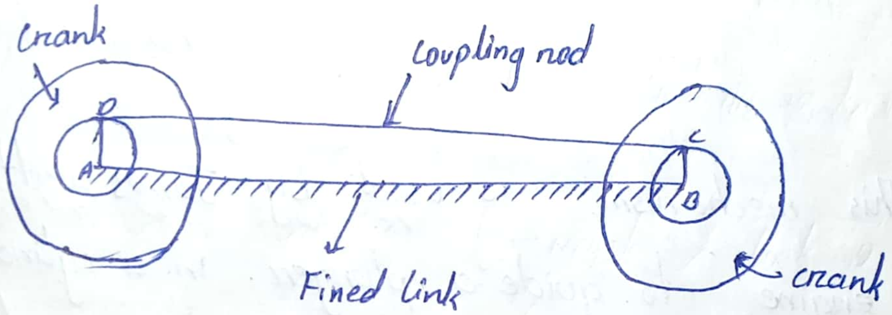
i) Link-1, (fixed link AF)   ii) Link-2 (crank)
iii) Link-3 (Connecting rod BC)   iv) Link-4 (Lever CE)



When we will give small displacement to link-2 or crank then the crank rotates in clockwise direction about centre point A. When the crank rotates, lever oscillates relative to link - BC at pivoted point D. So piston reciprocates inside the engine cylinder. this type of mechanism is used for transmitting rotary motion to reciprocating motion

Double crank mechanism

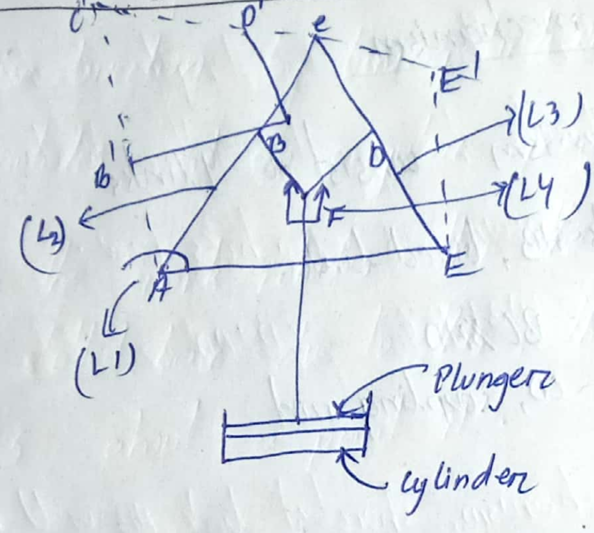
This mechanism consist of 4 links
i) Link-AB, fixed link
ii) Link - BC & AD
iii) Link - CD, coupling rod



When a small displacement is given to link-AD, crank rotates in anti-clockwise direction. when crank rotates, wheel, (n) rotates about it's centre point A & simultaniously the coupling rod rotates w.r.t crank AD.

Due to this, the crank-BC also rotates in anticlockwise direction and wheel is also rotates. this mechanism is used for transmitting rotary motion from one wheel to other wheel.

Watt indicator mechanism



This mechanism is used by James watt in steam engine to guide a plunger in a cylinder in order to have straight line motion.

→ This mechanism is also called as watt's straight line mechanism.

→ This mechanism permits only relative motion of an oscillating type link along a straight line.

It consists of 4-links

i) Link-A (fixed link)

ii) Link-AC (motion link)

iii) Link-CE (Lever)

iv) Link BFD (Lever)

By fixing link-A, when gases exerts pressure on the plumber. then the link BFD make a small displacement. So link AC displaced to small position.

- Therefore link-CE traces out as approximately straight line.

Q. Crank & slotted lever with quick return motion

- This mechanism is used in shaper machine
- In this mechanism, crank & slotted lever is used
- This mechanism is used for transmitting rotary motion into reciprocating motion.
- This mechanism consists of 4-links.

1) Link-1 ⇒ Slider
2) Link-2 ⇒ Crank (BC)
3) Link-3 ⇒ Fixed Link (AC)
4) Link-4 ⇒ slotted bar (AP)

When we will give small displacement to the crank in clockwise direction by bining link 'AC', then the crank rotates at an angular speed at point 'C'.

When the crank rotates then the slider slides along slotted bar due to this, the slotted bar oscillates about a point 'A'.

Then the rod PR transmitting motion to the RAM which carries a tool & this reciprocates along the line of stroke R_1 & R_2

The cutting stroke or, forward stroke takes place when the crank moves CB_1 to CB_2 in clockwise direction or the crank moves through an angle of β.

The backword or, return stroke takesplace when the crank moves from CB_2 to CB_1 in clockwise direction. or the crank moves an angle of α.

Since the crank rotate an uniform angular speed therefore

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{360-\alpha}{\alpha}$$

$$\Rightarrow \frac{\beta}{360-\beta}$$

→ There are three turning pair occurs i.e

link - 3-2 (bined link & crank)
link - 3-4 (bined link & slotted bar)
link - 1-2 (slider & crank)
and one sliding pair link 1-4

* Friction :- It is an opposing force which acts in the opposite direction of force.

Types of friction :-

In general the friction is of two types :-

1) Static friction :- It is the friction experienced by the body when at rest.

2) Dynamic friction :- It is the friction experienced by the body when in motion.

* The dynamic friction is also known as kinetic friction.

* It is of 3 types

i) Sliding friction :- It is the friction experienced by body when it slides over another body.

ii) Rolling friction :- It is the friction experienced betn the surfaces which has balls or rollers interposed betn them.

iii) Pivot friction :- It is the friction experienced by the body due to the motion of rotation as in case of foot step bearing.

SCREW FRICTION

The friction which is experienced in a screw thread, nuts, bolts, shafts, etc. is known as screw friction.

If the threads are cut on the outter surface of the solid rod then the threads are known as external threads. But, If the threads are cut on the internal surface of the hollow rod then they are known as internal threads.

Termenalogy used in Screw friction

1. Helix :-

It is the curve traced by the particle while describing a circular path which is advanced axially at a uniform rate. In other words it is the curve the traced by particle while moving along a screw thread.

2. Pitch

It is the distance from a point of a screw to corresponding point on the next point measured parallel to the axis of the screw.

3. Lead :- It is distance a screw thread advanced axially in one turn.

4. Depth of thread :- It is the distance betn the top and bottom surface of the thread. The top surface is known as crest and the bottom surface is root.

5. Single thread screw :- If the lead of a screw is equal to it's pitch, it is known as single thread -ed screw.

6. Multi-threaded screw :- If more than one thread is cut in one lead distance of a screw then it is known as multi-threaded screw.

Mathematically,

$$\text{Lead} = \text{Pitch} \times \text{No of threads.}$$

7. Helix Angle :-

It is the slope or the inclination of the thread with the horizontal.

Mathematically,

$$\tan\alpha = \frac{\text{lead of screw}}{\text{circumference of screw}}$$

$$= \frac{p}{\pi d}$$

$$= \frac{np}{\pi d} \text{ (for multi thread screw)}$$

where, α = Helix angle

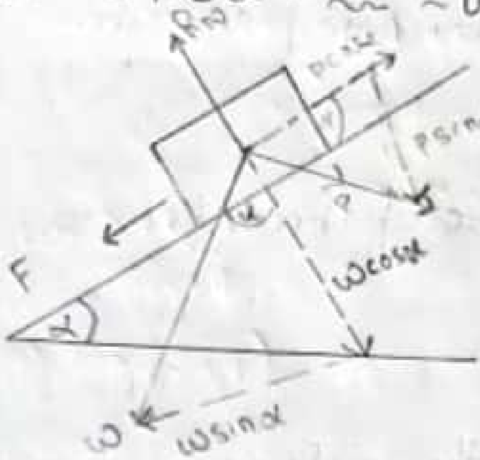
p = Pitch of screw

d: mean diameter of screw
n: no of threads in one lead.

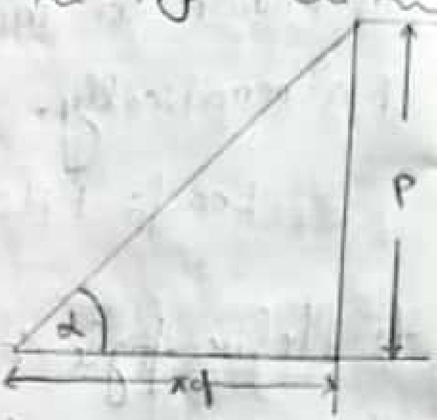
Screw Jack

A screw jack is a device for lifting heavy loads by applying comparatively smaller effort at the handle.

Torque Required to lift the load by a Screw Jack



(force on a screw)



(Development of screw)

Let, P = pitch of the screw
d = mean diameter of the screw
α = Helix angle
P: effort applied on the circumference of the screw to lift load.
W = load to be lifted
μ: co-efficient of friction betn the screw & the nut

$\mu = \tan \phi$

where, ϕ is the friction angle

From geometry of the fig we know that

$\tan\alpha = \frac{P}{\pi d}$

Resolve the force along the plane.

$P\cos\alpha = F + w\sin\alpha$

Resolve force vertically

$R_N = w\cos\alpha + P\sin\alpha$

$P\cos\alpha = w\sin\alpha + F$

$= w\sin\alpha + \mu R_N$ — (i)

$R_N = P\sin\alpha + w\cos\alpha$ — (ii)

Substitute the value of R_N in eqn - (i)

$P\cos\alpha = w\sin\alpha + \mu(P\sin\alpha + w\cos\alpha)$

$= w\sin\alpha + \mu P\sin\alpha + \mu w\cos\alpha$.

or $P\cos\alpha - \mu P\sin\alpha = w\sin\alpha + \mu w\cos\alpha$

$P(\cos\alpha - \mu\sin\alpha) = w(\sin\alpha + \mu\cos\alpha)$

$P = w \times \frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha + \mu\sin\alpha}$

substituting value of $\mu = \tan\phi$ in above eqn we get

$P = w \times \frac{\sin\alpha + \tan\phi\cos\alpha}{\cos\alpha + \tan\phi\sin\alpha}$

$$P = w \times \frac{\mu \cos\alpha + \sin\alpha}{\cos\alpha - \mu \sin\alpha}$$

$$= w \times \frac{\sin\alpha + \tan\phi \cdot \cos\alpha}{\cos\alpha - \tan\phi \cdot \sin\alpha}$$

$$= w \times \frac{\sin\alpha \cdot \cos\phi + \sin\phi \cdot \cos\alpha}{\cos\alpha \cdot \cos\phi - \sin\phi \cdot \sin\alpha}$$

$$= w \times \frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi}$$

$$= w \tan(\alpha+\phi)$$

Torque required, required to overcome friction at the collar.

$$T_2 = \mu_1 \times w \left(\frac{R_1 + R_2}{2} \right)$$

$$= \mu_1 \times w \times R$$

where, R_1 & R_2 = outside and inside Radii of the collar.

r_c = mean radius of collar.

μ_1 : co-efficient of friction for the collar

Therefore, torque required to overcome the friction

$T = T_1 + T_2$

$= P \times \frac{d}{2} + \mu_1 W R$

If an effort P_1 is applied at the end of a lever of arm length l, then the total torque required to overcome friction must be equal to the total torque applied at the end of the lever.

$T = P_1 \times \frac{l}{2} = P_1 \times l$

Note:- 1) when the nominal dia (d_0) and the core dia (d_c) of the screw thread is given then the mean dia of the screw

$$d = \frac{d_0 + d_c}{2} = d_0 - \frac{P}{2} \times d_c + \frac{d}{2}$$

2) Since the Mechanical Advantage is the ratio of load lifted (W) to the effort applied (P_1) at the end of the lever therefore,

$$M.A = \frac{W}{P_1}$$

$$= \frac{W \times 2l}{Pd}$$

$$= \frac{W \times 2l}{W \tan(\alpha+\phi) \pi d}$$

$$\boxed{M.A. = \frac{2l}{d \tan(\alpha+\phi)}}$$

Problems...

An electric motor driven screw moves a nut in a horizontal plane against the force of 75 kN at a speed of 300 mm/min. The screw has single sq thread of 6mm pitch on a measure dia of 40mm. The co-efficient of frion at the screw thread is 0·1. Estimate power of the motor.

Given :- $W = 75\,kN$

$d_0 = 40$

$P = 6\,mm$

$\mu = 0.1$

$N = 300\,mm/min$

$N = \frac{300}{6}\ rpm = 50\ rpm$

$P = \frac{2\pi N T}{60}$

$T = P \times \frac{d}{2}$

$P = W \tan(\alpha+\phi)$

1.108 kN

$= \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \cdot \tan\phi}$

$\tan\alpha = \frac{p}{\pi d}$

$d = d_0 - \frac{p}{2}$

$\boxed{d = 37}$ / 37mm

$\tan\alpha = \frac{6}{\pi \times 37}$

$\tan\alpha = 0.05161$

$\tan\phi = \mu = 0.1$

$\tan\phi = 0.1$

$P = W\tan(\alpha + \phi)$

$= W\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha - \tan\phi}$

$= 11.42\ N$

~~$T = P \times \frac{d}{2}$~~

~~$= \times \frac{37}{2}$~~

~~$= 2.817$~~

~~$P = \frac{2\pi \times 60 \times 2.817}{6}$~~

P_2

$T = 11.42 \times 10^3 \times \frac{37}{2}$

$= 211.27\ N\text{-}m$

$P = \frac{2\pi NT}{60}$

$\left(N = \frac{\text{Speed of the nut}}{\text{Pitch of the screw}} \right)$

$= \frac{2 \times \pi \times 50 \times 211.27}{60}$

$= 1.106\ kN$

Q; A square threaded bolt of root diameter 22.5 mm & pitch 5mm is tightened by screwing a nut whose mean diameter of bearing surface is 30mm. If coefficient of friction for nut & bolt is 0.1 & coefficient of friction for nut & bolt is 0.1 & for nut & bearing surface is .16. Find the force required at the end of the spanner 500mm long when the load on the bolt is 10kN.

Ans Given:- $d_c = 22.5mm$

Pitch 'p' = 5mm

$D = 50mm$

$\mu = 0.1$

$W = 10kN = 10 \times 10^3 N$

$\mu_1 = .16$

$l = 50mm$

$d = d_c + \frac{p}{2}$

$= 22.5 + \frac{5}{2}$

$= 25mm$

$\tan\alpha = \frac{P}{\pi d}$

$= \frac{5}{\pi \times 25}$

$= 0.0636$

$P = W\tan(\alpha+\phi)$

$W \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \cdot \tan\phi}$

$= 10 \times 10^3 \times \frac{0.0636 \times 10^{-3} \times .1}{1 - 0.0636 \times 10^{-3} \times 0.1}$

$= 1646N$

$T = P \times \frac{d}{2} + \mu_1 WR$

$= 1646 \times \frac{25}{2} + .16 \times 10 \times 10^3 \times \frac{50}{2}$

$= 60575 Nmm$

$T = P_1 \times l$

$P_1 = \frac{T}{l}$

$= \frac{60575}{50}$

$= 1211.5$

Q) A 150mm Dia valve against which a steam pressure of 2 N/mm2 is acting is closed by mean of a square threaded screw 50mm external diameter with 6mm pitch. If the co-efficient of friction is 0.17, find the torque required to turn the handle.

A) Dia valve 'D' = 150mm
= 0.15m

Pitch 'p' = 6mm

d_o = 50mm

μ = .12

Steam pressure = 2 N/mm2

$= 2 \times 10^{6}$ MN/m2

~~W = P×A~~ W = P×A

$= 2 \times 10^6 \times \frac{\pi}{4} \times 2$

= 35200 N

= 35.2 kN

$d = d_o - \frac{P}{2}$

$= 50 - \frac{6}{2}$

= 47mm

$\tan\alpha = \frac{P}{\pi d}$

$= \frac{6}{\pi \times 47}$

= .0406

$P = W \tan(\alpha + \phi)$

$= W \dfrac{\tan\alpha + \tan\phi}{1 - \tan\alpha \cdot \tan\phi}$

$= 35200 \times \dfrac{0.04086 + 0.12}{1 - 0.0408 - 0.12}$

$= 5680.79$

$T = P \times \dfrac{d}{2}$

$= 5680.79 \times \dfrac{47}{2}$

$= 133498.5$

$= 133451$ N-mm

TORQUE REQUIRED TO LOWER THE LOAD BY SCREW JACK :-

From geometry of fig we find that $\tan\alpha = \dfrac{P}{\pi d}$

Resolve the forces along the plane

$P\cos\alpha = F - W\sin\alpha$

$P\cos\alpha = \mu R_N - W\sin\alpha$ —— (i)

Resolve the forces ⊥r to the plane

$P\cos R_N = W\cos\alpha - P\sin\alpha$ —— (ii)

Substitute the value of R_N in eqn. (i)

$$P\cos\alpha = \mu f(W\cos\alpha - P\sin\alpha) - W\sin\alpha$$

$$= \mu f W\cos\alpha - \mu f P\sin\alpha - W\sin\alpha$$

$$P\cos\alpha + \mu f P\sin\alpha = \mu f W\cos\alpha - W\sin\alpha$$

$$P(\cos\alpha + \mu f\sin\alpha) = W(\mu f\cos\alpha - \sin\alpha)$$

$$P = W\frac{(\mu f\cos\alpha - \sin\alpha)}{\cos\alpha + \mu f\sin\alpha}$$

Substitute the value of $\mu f = \tan\phi$ in the above eqn

$$P = W\frac{(\tan\phi\cos\alpha - \sin\alpha)}{\cos\alpha + \tan\phi\sin\alpha}$$

Multiply the numerator & denominator by $\cos\phi$

$$P = \frac{W(\sin\phi\cos\alpha - \sin\alpha \times \cos\phi)}{\cos\phi\cos\alpha + \sin\phi\sin\alpha}$$

$$P = W\frac{\sin(\phi-\alpha)}{\cos(\phi-\alpha)}$$

$$= W\tan(\phi-\alpha$$

Torque required to over-come the friction betn the screw & nut

$$T = P \times d/2$$

$$= W\tan(\phi-\alpha) \times d/2$$

NOTE :- when α is greater than ϕ

$$P = W\tan(\alpha-\phi)$$

EFFICIENCY OF A SCREW JACK :-

The efficiency of a screwjack may be defined as the ratio betn ideal effort (i.e the effort required to move the load neglecting friction) to the actual load effort (i.e, the effort required to move the load considering the friction).

We know that,

Effort required to move the load when friction is considered

$$P = W \tan(\alpha + \phi)$$

when friction is considered then the value of ϕ becomes zero.

Therefore the actual effort $P_0 = W \tan\alpha$

$\therefore$ efficiency $(\eta) = \frac{\text{Ideal}}{\text{actual}} = \frac{W \tan\alpha}{W \tan(\alpha+\phi)} = \frac{P_0}{P}$

Maximum Efficiency Of Screw Jack :-

$$\eta = \frac{\tan\alpha}{\tan\alpha+\phi} = \frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{\sin\alpha+\phi}{\cos\alpha+\phi}}$$

$$= \frac{\sin\alpha}{\cos\alpha} \times \frac{\cos\alpha+\phi}{\sin\alpha+\phi}$$

$$= \frac{\sin\alpha \cdot \cos(\alpha+\phi)}{\cos\alpha \cdot \sin(\alpha+\phi)}$$

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Multiply denominator & numerator by 2

$$= \frac{2\times \sin\alpha \cdot \cos(\alpha+\phi)}{2\times \cos\alpha \cdot \sin(\alpha+\phi)}$$

$$= \frac{\sin(\alpha+\alpha+\phi) + \sin(\alpha-\alpha-\phi)}{\sin(\alpha+\alpha+\phi - \sin(\alpha-\alpha-\phi)}$$

$$= \frac{\sin(2\alpha+\phi) - \sin\phi}{\sin(2\alpha+\phi) + \sin\phi}$$

The efficiency should be maxm when $(2\alpha+\phi)$ is maxm

$$\therefore \sin(2\alpha+\phi) = 1$$

$$2\alpha+\phi = 90^\circ$$

$$2\alpha = 90-\phi$$

$$\alpha = 90-\phi$$

$$\alpha = 45-\frac{\phi}{2}$$

Substitute the value of 2α in above eqn

$$\frac{\sin(90-\cancel{\phi}+\cancel{\phi}) - \sin\phi}{\sin(90-\cancel{\phi}+\cancel{\phi}) + \sin\phi}$$

$$= \frac{\sin 90 - \sin\phi}{\sin 90 + \sin\phi}$$

$$\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}$$

Q) The mean dia of a square threaded screw jack is 50mm. The pitch of the thread is 10mm. The coefficient of friction is 0.15. what force must be applied at the end of a 0.7m long lever which is ⊥r to longitudinal axis of the screw to raise a load of 20kN & to lower it.

A) $d = 50mm$

$P = 10mm$

$\phi = 0.15mm$

$W = 20kN$

$l = .7m$

$\tan\alpha = \frac{P}{\pi d}$

$= 0.636$

$P = W\tan(\alpha+\phi)$

$= W \times \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \cdot \tan\phi}$

$= 20\times10^3 \times \frac{0.0636 + .15}{1 - 0.0636 \times 0.15}$

$= 4314 N$

$P_1 = \frac{P \times d}{2l}$

$= \frac{4314 \times 50 \times 10^{-3}}{2 \times 0.7}$

$= 154N$ (Raise)

$P = W\tan(\phi - \alpha)$

$= \omega \times \frac{\tan\phi - \tan\alpha}{1 + \tan\phi \cdot \tan\alpha}$

$= 20\times10^{3} \times \frac{0.15 - 0.0836}{1 + 0.15 \times 0.0636}$

$= 1709.6\ N$

$P_1 = \frac{Pd}{2l}$

$= \frac{1709 \times 50\times10^{-3}}{2\times .7}$

$= 61 N$.(lower)

Q) The pitch of 50mm. mean diameter threaded screw of a screw jack is 12.5 mm. The co-efficient of friction betn the screw & nut is 0.13. Determine the torque required on the screw to raise a load of 25kN ai swing the load to rotate with the screw. Determine the ratio of the torque required to raise the load & torque required to lower the load & also efficiency of the machine?

A) Given :-

$\omega = 25kN$

$d = 50mm = .05m$

$\phi = .13$

$P = 12.5mm$

$= .0125m$

$\tan\alpha = \frac{P}{\pi d} = .0795$

$P = \omega \tan(\alpha + \phi)$

$= \omega \times \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha - \tan\phi}$

$$= 25\times10^3 \times \frac{0.795 + 0.13}{1 - 0.795\times0.13}$$

$$= 5292\ N$$

$$P = w\tan(\phi - \alpha)$$

$$= w \times \frac{\tan\phi - \tan\alpha}{1+\tan\phi \cdot \tan\alpha}$$

$$= 25\times10^3 \times \frac{0.13 - .795}{1+ .13 - .795}$$

$$= 1246.6\ N$$

$$T_1 = P\times d/2$$

$$= 5292 \times 50/2$$

$$= 132300$$

$$\frac{T_1}{T_2} = \frac{132300}{31150}$$

$$= 4.24$$

$$T_2 = P\times d/2$$

$$= 1246 \times 50/2$$

$$= 31150$$

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \frac{\tan\alpha}{\frac{\tan\alpha+\tan\phi}{1-\tan\alpha\cdot\tan\phi}}$$

$$= 0.3767$$

$$= 37.67$$

Q. A Load of 10kN is raised by the means of a screw jack having a square threaded screw of 12mm pitch & mean dia of 50mm.

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If a force of 100N is applied at the end of the lever to raise the load, what should be the length of the lever is to used? Take ϕ = ·15. what is M·A obtained? state whether the screw is self locking or over hauling.

A) Given:- $W = 10kN$, $P = \cdot 012m$ $d = \cdot 05m$ $P_1 = 100N$ $\phi = \cdot 15$
$l = ?$

$\tan\alpha = \frac{P}{\pi d} = \cdot 0763$

$P = W \tan(\alpha + \phi)$

$= W \times \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha - \tan\phi}$

$= 2289 \cdot 1 N$

$P_1 \times l = P \times d/2$

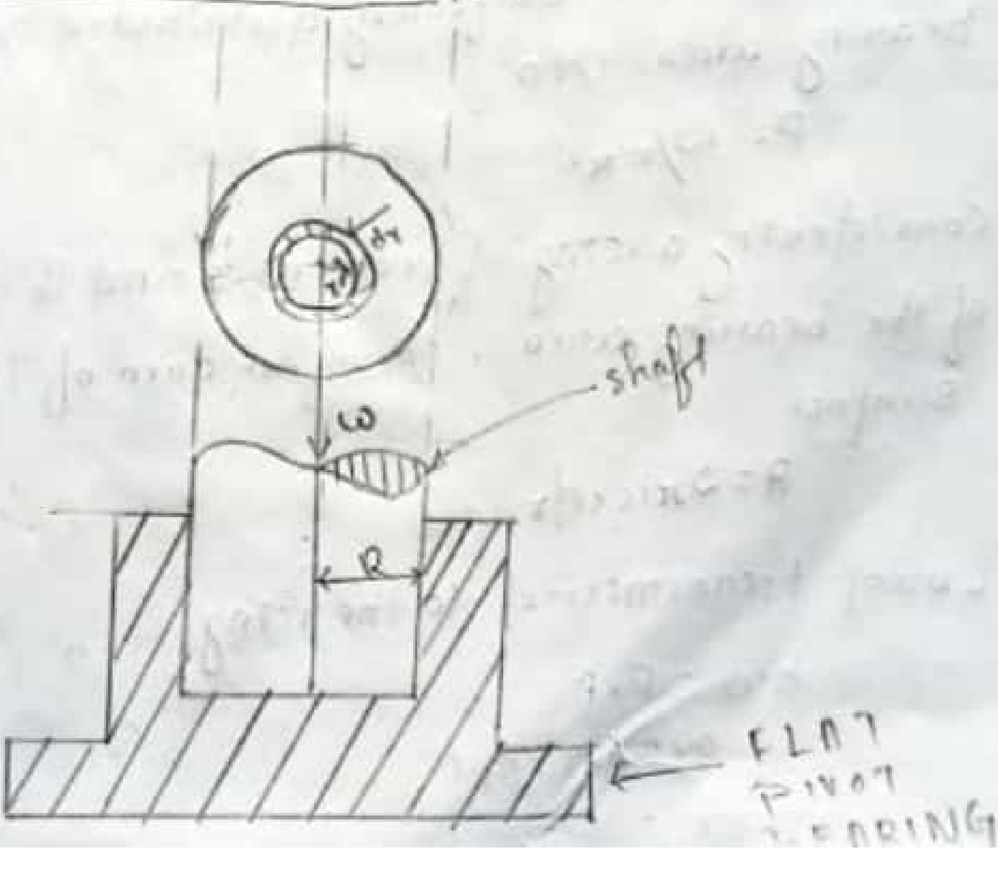
$l = \frac{Pd}{P_1 \times 2} = \frac{2289 \times \cdot 05}{100 \times 2}$

$= \cdot 5722m = 572 \cdot 2mm$

$M \cdot A = \frac{W}{P_1} = 100N$ $\eta = \cdot 3333 = 33 \cdot 33\%$.

As the η is less than 50% - the screw is self locking.

FLAT PIVOT BEARING :-



... vertical shaft rotates in a flat pivot bearing know as footstep bearing, this sliding friction will be along the surface of contact betn the shaft and the bearing surface.

let, W: Load transmitted over the bearing surface

R: radius of the bearing

P: Intensity of pressure per unit area of the bearing surface.

μ: coefficient of friction

Consider the following two cases:-

i) when there is a uniform pressure

ii) when there is a uniform wear

Case-I

Considering Uniform Pressure :-

When Pressure is uniformly distributed over the bearing area, then

$$P = W/\pi R^2$$

Considering a ring of radius r and thickness dr of the bearing area, therefore area of the bearing surface

$$A = 2\pi r \times dr$$

Load transmitted to the ring

$$\delta W = P \times A$$

$$= P \times 2\pi r \times dr$$

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let, W = Load transmitted over the bearing surface

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$$A = 2\pi r \times dr$$

Load transmitted to the ring

$$\delta W = P \times A$$

$$= P \times 2\pi r \times dr$$

Frictional resistance to sliding on the ring acting tangentially to radius r

$F_r = \mu \times \delta w$

$= \mu \times P \times 2\pi r\, dr$

Frictional torque on the ring

$T_r = F_r \times r$

$= \mu \times P 2\pi r\, dr \times r$

$T_r = \mu P \times 2\pi r^2 dr$

Integrating the eqn within the limits from 0 to R for the total torque on the pivot bearing

$T = \int_0^R \mu P 2\pi r^2 dr$

$= \mu P 2\pi \int_0^R r^2 dr$

$= \mu P\, 2\pi R^3/3$

$T = 2/3\, \pi \mu P R^3$

Substituting the value of P in this eqn

$= 2/3\, \cancel{\pi} \times \mu \times \frac{W}{\cancel{\pi} R^2} \times R^{\cancel{3}}$

$T = 2/3\, \mu W R$

Case-II

Considering uniform wear

It is assumed that the rate of wear is proporti
to the product of intensity of pressure & velo
of the rubbing surface. Since the velocity of
rubbing surface increases with the distance fr
the axis of the bearing, therefore uniform wea

$$P \times r = C$$

$$P = \frac{C}{r}$$

Load transmitted on the ring

$$\delta w = P \times 2\pi r\,dr$$

$$= \frac{C}{\cancel{r}} \times 2\pi \cancel{r}\,dr$$

$$= 2\pi C\,dr$$

Total load transmitted on the bearing surfa

$$w = \int_0^R 2\pi C\,dr$$

$$= 2\pi C \int_0^R dr$$

$$= 2\pi C \left[r\right]_0^R$$

$$w = 2\pi C R$$

$$C = \frac{w}{2\pi R}$$

we know that the frictional torque acting on the ring

$$T_r = 2\pi\eta f \cdot P r^2 dr$$

$$= 2\pi r \eta \, c/y \, r \, dr$$

$$= 2\pi \cdot r \cdot c r \, dr$$

Now the total frictional torque on the bearing surface.

$$T = \int_0^R 2\pi \eta f \, c r \, dr$$

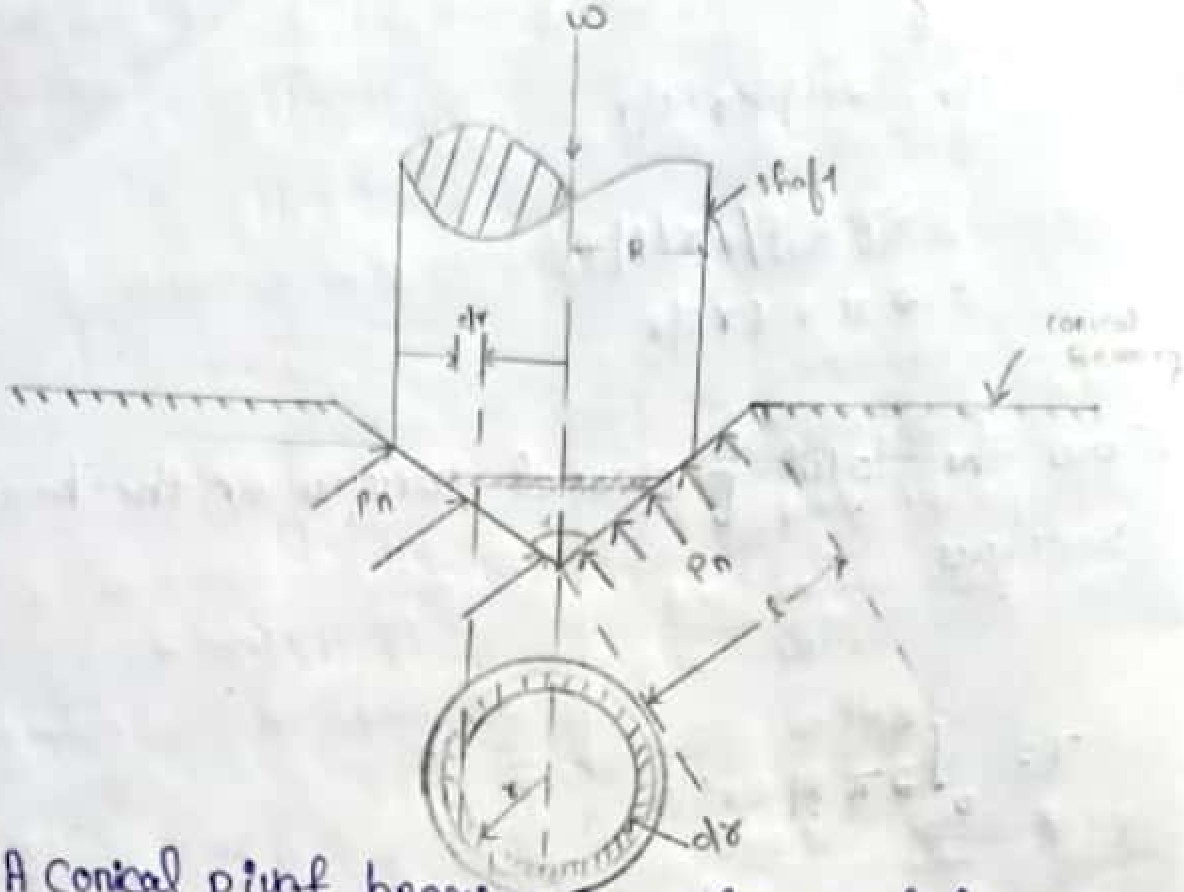
$$= 2\pi\eta f c \int_0^R r \, dr$$

$$= 2\pi\eta f \, c R^2$$

$$= 2\pi \eta f \frac{\omega}{2\pi R} \times R^2$$

$$= \frac{1}{2} \eta f \omega \cdot R.$$

Conical Pivot Bearing :-



A conical pivot bearing supporting a shaft carrying a load 'W' as shown in figure.

Let,

P_n = Intensity of pressure normal to the cone
α = Semi-angle of the cone
μ = Coefficient of friction betn shaft & bearing
R = Radius of the shaft

Consider a small ring of radius 'r' and thickness dr.

Let, dl = length of the ring along the cone such that

$dl = dr \csc\alpha$

$\therefore$ area of the ring $A = 2\pi r \times dl$

$= 2\pi r \times dr \operatorname{cosec}\alpha$ ($\because dl = dr \operatorname{cosec}\alpha$)

Case - I Considering Uniform Pressure:

• we know that normal load acting on the ring

$\delta W_n = P_n \times \text{Area}$

$= P_n \times 2\pi r \, dr \operatorname{cosec}\alpha$

• Vertical load on the ring

$\delta W = $ Vertical component of δW_n

$\delta W = \delta W_n \times \sin\alpha$

$= P_n \times 2\pi r \, dr \operatorname{cosec}\alpha \cdot \sin\alpha$ ($\because \operatorname{cosec}\alpha \cdot \sin\alpha = 1$)

$= P_n \times 2\pi r \, dr$

Total vertical load transmitted to bearing

$$W = \int_0^R P_n \times 2\pi r \cdot dr$$

$$= P_n \times 2\pi \int_0^R r \, dr$$

$$= P_n \times 2\pi \left[\frac{r^2}{2}\right]_0^R$$

$$= 2\pi P_n \frac{R^2}{2}$$

$$W = \pi P_n R^2$$

$$P_n = \frac{W}{\pi R^2}$$

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• we know that frictional force acting on the ring acting tangentially at radius r

$F_r = \mu \times \delta W_n$

$F_r = \mu \times P_n \times 2\pi r\, dr \times \operatorname{cosec}\alpha$

Frictional Torque acting on the ring

$T_r = F_r \times r$

$= \mu \times P_n \times 2\pi r\, dr \times \operatorname{cosec}\alpha \times r$

$2\pi \mu P_n \operatorname{cosec}\alpha\, r^2 dr$

The total torque can be found out by integrating the above expression from limit 0 to R.

$$\int_0^R 2\pi \mu P_n \operatorname{cosec}\alpha\, r^2 dr$$

$$= 2\pi \mu P_n \operatorname{cosec}\alpha \int_0^R \frac{r^3}{3}$$

$$= 2\pi \mu P_n \operatorname{cosec}\alpha \left[\frac{r^3}{3}\right]_0^R$$

$$= 2\pi \mu P_n \operatorname{cosec}\alpha \frac{R^3}{3}$$

$$= \frac{2}{3}\pi \mu P_n \operatorname{cosec}\alpha\, R^3 \quad \text{--- (1)}$$

Substitute the value of P_n in eqn 1 is

$$\frac{2}{3}\pi\mu \frac{W}{\pi \cancel{R^2}} \operatorname{cosec}\alpha\, \cancel{R^3}$$

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$= \frac{2}{3}\pi \mu \frac{W}{\pi} \operatorname{cosec}\alpha\, R$

$= \frac{2}{3}\mu W \operatorname{cosec}\alpha\, R$

$= \frac{2}{3}\mu W R \operatorname{cosec}\alpha$

$\boxed{T = \frac{2}{3}\mu W l}$ $\quad \because \boxed{l = R \operatorname{cosec}\alpha}$

Case-II

Considering uniform wear:

Let,

P_r = Normal intensity of pr. at a distance r from the center axis

$P_r \times r$ = Constant (c)

$P_r = \frac{c}{r}$

Load transmitted to the ring.

$\delta w = P_r\, 2\pi r \times dr$

$\delta w = \frac{c}{\cancel{r}} 2\pi \cancel{r}\, dr$

$\delta w = 2\pi c\, dr$

Total load transmitted to the ring

$W = \int_0^R 2\pi c\, dr$

$= 2\pi c \int_0^R dr$

$= 2\pi c R$

$c = \frac{W}{2\pi R}$ ——— (1)

Frictional torque transmitted to ring

$T_r = F_r \times r$

$= \mu P_r 2\pi r \, dr \operatorname{cosec}\alpha \times r$

$= \mu \frac{c}{\cancel{r}} 2\pi \cancel{r} \, dr \operatorname{cosec}\alpha \times r$

$= 2\pi \mu C \operatorname{cosec}\alpha \, r \, dr$

Total torque (T)

$= \int_0^R 2\pi \mu C \operatorname{cosec}\alpha \, r \, dr$

$= 2\pi \mu C \operatorname{cosec}\alpha \int_0^R r \, dr$

$= 2\pi \mu C \operatorname{cosec}\alpha \frac{r^2}{2}$

$= \pi \mu C \operatorname{cosec}\alpha \left[r^2\right]_0^R$

$= \pi \mu C \operatorname{cosec}\alpha \, R^2$

$= \cancel{\pi} \mu \frac{\omega}{2\cancel{\pi} \cancel{R}} \times \operatorname{cosec}\alpha \, R^{\cancel{2}}$ (substitute the value of c)

$= \frac{1}{2} \mu \omega \operatorname{cosec}\alpha \, R$

$\boxed{T = \frac{1}{2} \mu \omega l}$ $(\because \operatorname{cosec}\alpha \, R = l)$

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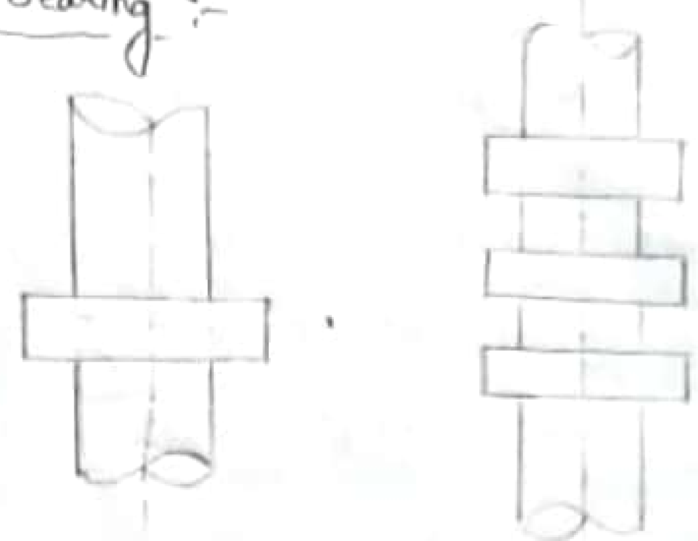
Flat colar bearing :-



Consider a single flat colar bearing supporting a shaft as shown in fig.

r_1 = External radius of the colar
r_2 = Internal " " " "

Ar of the bearing surface $= \pi[(r_1)^2 - (r_2)^2]$

Case-1 :- Considering uniform pressure

Intensity of pressure

$$P = \frac{W}{A}$$

$$= \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]}$$

As we already know the frictional torque on the ring of radiation & thickness of the ring.

$$T_r = 2\pi\mu P r^2 dr$$

Integrating the eqn with in the limit r_2 to r_1 the to total frictional torque

$$T = \int_{r_2}^{r_1} 2\pi\mu \cdot P r^2 dr$$

$$= 2\pi\mu P \int_{r_2}^{r_1} r^2 dr$$

$$= 2\pi\mu P \int_{r_2}^{r_1} \frac{r^3}{3}$$

$$= 2\pi\mu P \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu P \left[\frac{(r_1)^3 - (r_2)^3}{3}\right] \quad \text{--- (1)}$$

Substitute the value of "P" in eqn -1

$$T = 2\pi\mu \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]} \times \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$T = \frac{2}{3} \mu w \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

Note:- i) In case of a multi collar bearing say "n" no of collars then the intensity of uniform pressure

$$P = \frac{W}{n\pi \left[(r_1)^2 - (r_2)^2 \right]}$$

ii) The total torque transmitted in collar bearing remains constant.

i.e : $T = \frac{2}{3} \mu w \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$

Case - 2 Considering uniform wear.

The load transmitted in the ring considering uniform wear.

$$\delta w = P_r \times 2\pi r \, dr$$

$$= \frac{c}{\cancel{r}} \times 2\pi \cancel{r} \, dr$$

$$= 2\pi c \, dr$$

Total load transmitted to the torque

$$w = \int_{r_2}^{r_1} 2\pi c \, dr$$

$$2\pi c \int_{r_2}^{r_1} dr = 2\pi c (r_1 - r_2)$$

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$$c = \frac{W}{2\pi(r_1 - r_2)}$$

The frictional torque on the ring

$T_r = \mu \times \delta W \times r$

$T_r = \mu \times 2\pi c \times r\,dr$

Total frictional torque on bearing

$$T = \int_{r_2}^{r_1} 2\pi \mu c\, r\,dr$$

$$= 2\pi \mu c \int_{r_2}^{r_1} r\,dr$$

$$= 2\pi \mu c \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= \cancel{2}\pi \mu c \left[\frac{(r_1)^2 - (r_2)^2}{\cancel{2}}\right]$$

$$= \pi \mu c \left[(r_1)^2 - (r_2)^2\right] \quad \text{--- (i)}$$

Substitute the vale of c in eqn-(i)

$$T = \cancel{\pi} \mu \times \frac{W}{2\cancel{\pi}(r_1 - r_2)} \left[(r_1)^2 - (r_2)^2\right]$$

$$T = \mu \times \frac{W}{2(r_1 - r_2)} \left[(r_1 + r_2)(r_1 - r_2) \right]$$

$$T = \frac{1}{2} \times \mu W (r_1 + r_2)$$

A thrust shaft of the ship has a 6 collars of 600mm external dia & 300 mm internal dia. The total thrust from the propular is 100 kN. If coefficient of fricition is .12 and speed of engine is 90 ~~r.p.m~~ r.p.m. Find the power absorbed in friction at the thrust block. Assuming

(i) uniform pressure
(ii) uniform ~~power~~ wear.

$W = 100kN$

Given :- (r_1) External dia : 600mm or 0.3 m of radius
(r_2) internal dia : 300mm or 0.15 m of radius
$\mu = .12$

~~$T = 2\pi \mu P_r^2 dr$~~

$$Pr = \frac{F}{A} = \frac{100\,kN}{\pi (r_1^2 - r_2)^2}$$

384.255

i) In case of uniform pr

$$T = \frac{2}{3} \mu W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right]$$

$$= \frac{2}{3} \times .12 \times 100 \left[\frac{(.3)^3 - (.15)^3}{(.3)^2 - (.15)^2} \right]$$

$= 2.8$ kN-M or 2800 N-M

$$P = \frac{2\pi NT}{60}$$

$$P = \frac{2 \times \pi \times 90 \times 2800}{60}$$

$$\boxed{= 26.4\ kW}$$

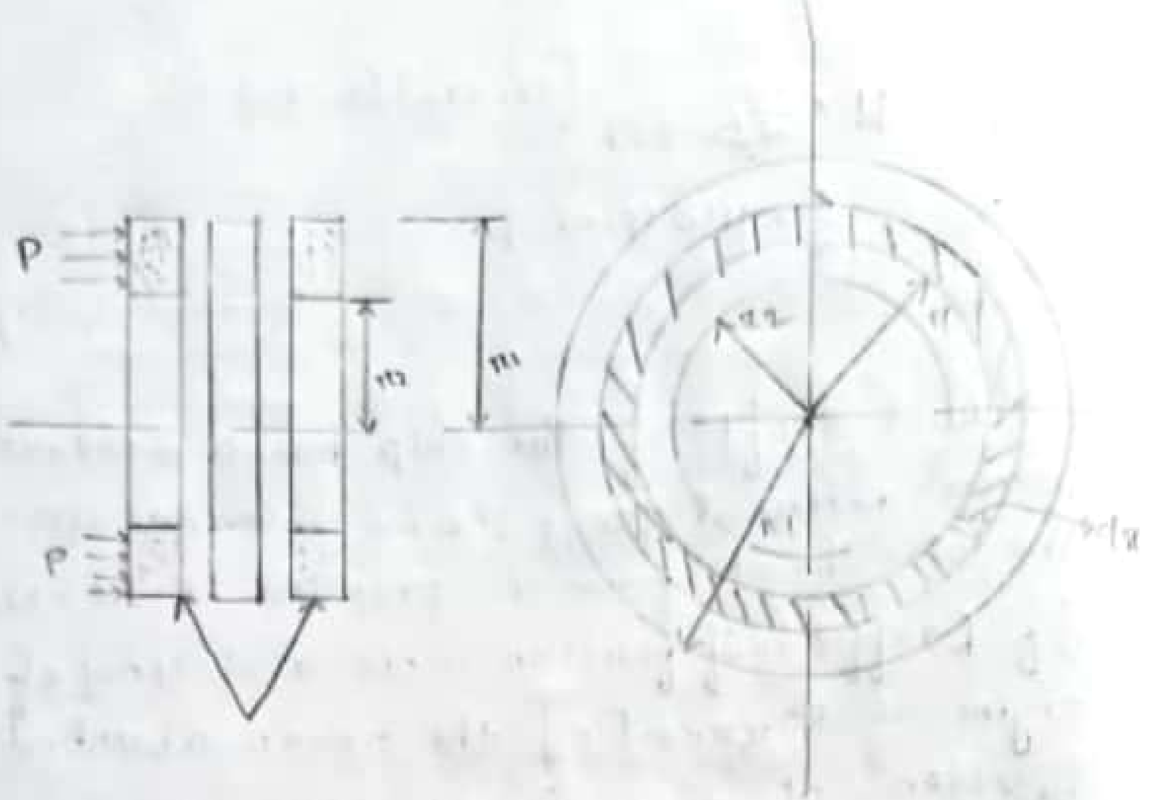
ii) In case of uniform wear

$= \frac{1}{2} \times \mu W (r_1 + r_2)$ ~~$2\pi \mu P_r^2 dr$~~

$= 2.7$ kN-m

$$P = \frac{2\pi NT}{60}$$

$$\boxed{= 25.45\ kW}$$



Let, t = torque transmitted by clutch

p = intensity of axial pressure with which the contact surface are held together

r_1 & r_2 = External & Internal radii of friction surface

μ = Coefficient of friction

Consider an elementary ring of radius r and thickness dr

we know that, ar. of contact surface of r friction surface $= 2\pi r \times A$
$= 2\pi r \, dr$

Normal force on the ring $\delta W = p \times A$
$= p \times 2\pi r \times dr$

Frictional force on the ring acting tangentiall at radius r

$F_r = \mu \times \delta W$

$= \mu \times p \times 2\pi r dr$

Frictional torque : $T_r = F_r \times r$

$= \mu \times p \times 2\pi r^2 dr$

Case-I

Considering uniform pressure

when the pr is uniformly distributed over the entire area of friction faces then the intensity of pressure

$$p = \frac{W}{\pi(r_1^2 - r_2^2)}$$

we already know that frictional torque

$T_r = 2\pi \mu p r^2 dr$

Integrating above eqn from limit r_2 to r_1

$$\int_{r_2}^{r_1} 2\pi \mu p r^2 dr$$

$$2\pi \mu p \int_{r_1}^{r_2} \frac{r^3}{3}$$

$$2\pi \mu p \left(\frac{r_1^3 - r_2^3}{3}\right) \quad \cdots (i)$$

Substituting the value of p in eqn-(i)

$\frac{2}{3}$

$= \mu W R$

where, R = mean radius of friction surface whose value is

$$R = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

<u>Case-2</u> :- <u>Considering uniform wear</u>

Let, p be normal intensity of pressure at distance r from axis of clutch

we know that, $$p r = C$$
$$p = \frac{C}{r}$$

Normal force on the ring

$$\delta w = p r \times 2\pi r \times dr$$
$$= \frac{C}{r} \times 2\pi r \times dr$$
$$= 2\pi C\, dr$$

Total frictional force

$$W = \int_{r_2}^{r_1} 2\pi C\, dr$$

$$W = 2\pi C(r_1 - r_2)$$

$$c = \frac{W}{2\pi(r_1 - r_2)}$$

we know that the frictional torque acting on the ring

$$T_r = 2\pi \mu P r^2 dr$$
$$= 2\pi \mu \frac{c}{r} \times r^2 dr$$
$$= 2\pi \mu c r \, dr$$

Total frictional torque

$$T = \int_{r_2}^{r_1} 2\pi \mu c r \, dr$$
$$= 2\pi \mu c \int_{r_2}^{r_1} r \, dr$$
$$= 2\pi \mu c \frac{r_1^2 - r_2^2}{2}$$
$$= \pi \mu c (r_1^2 - r_2^2)$$
$$= \pi \mu \frac{W}{2\pi(r_1 - r_2)} (r_1^2 - r_2^2)$$
$$= \mu \frac{W}{2\cancel{(r_1 - r_2)}} \cancel{(r_1 - r_2)} (r_1 + r_2)$$
$$= 2 \mu W (r_1 + r_2)$$

where $R = \cancel{r_1 + r_2}$ mean radius of the friction surface whose value is

Note :- In general total frictional torque acting on friction surface is given by

$$T = n \mu W R$$

where, n = no of pairs of friction surface
R = mean radius of friction surface

$= \frac{2}{3}$ (for uniform pressure)

$= \frac{r_1+r_2}{2}$ (for uniform wear)

2) For single plate clutch both side of the disc are effective. Therefore single disc clutch has 2 pairs of contact surface i.e $n=2$

3) Since, the intensity of pressure is maxm at the inner radius (r_2) therefore the eqn may be written as

$P_{max} \times r_2 = C$

4) Since, the intensity of pressure is minm at outter radius therefore the eqn may be written as

$P_{min} \times r_1 = C$

Multiple disc clutch

Let, n_1 = no of disc on driving shaft
n_2 = no of disc on driven shaft

therefore, no of pairs of contact surface of

$n = n_1 + n_2 - 1$

Total frictional torque

$T = n\mu WR$

Power transmission

Belt

→ Belt are used to transmit power from one saft to another with the help of pully.

→ The other methods of transmitting powers are rope & chain

→ Belt, rope is used where the distance betn the saft is large.

→ Gears are used where the distance is small.

Classification of belt drive

It is classified in to 3 types

i) Light belt drive
ii) Medium belt drive
iii) Heavy belt drive

1) Light belt drive :- when the belt speed is 0-10 m/s, then it is called light belt drive

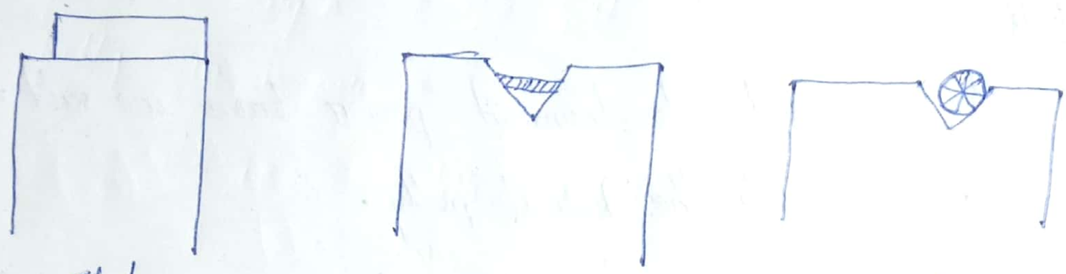
2) Medium belt drive :- when the belt speed is 10-22 m/s then it is called medium belt drive.

3) Heavy belt drive :- when the belt speed is more then 22 m/s then it is called heavy belt drive.

Types of belts

It is of 3 types

1) Flat belt
2) V- belt
3) [illegible]



→ Flat belt drive is rectangular in cross-section
→ V-belt drive is trapizodal in cross-section
→ Circular belt drive is circular in cross-section.

Material of belt

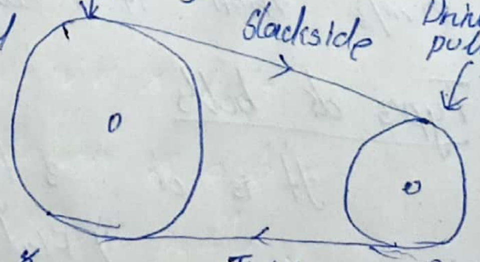
:- leather, cotton or fabric, rubber

Types of Flat belt drive

It is classified in to following types
i) Open belt drive
ii) Cross belt drive
iii) Compound belt drive

Open belt drive

→ The open belt drive is used with the ~~sa~~ shaft arranged in parallel and rotating in same direction.

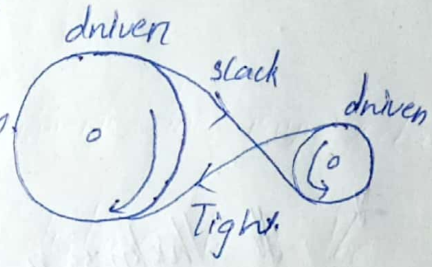


→ In this case, the driver pully pulls the belt from one side & delivers it to other side. thus tension in the lower side is more then the upper side

→ Therefore the lowerside belt is called tight side & upper side is called slack side.

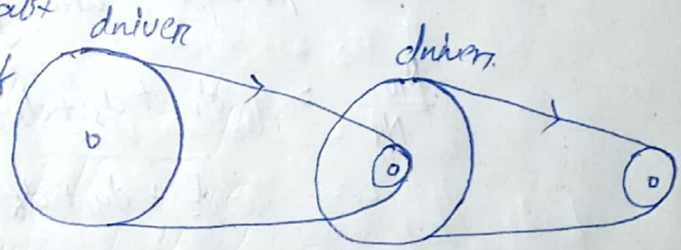
Cross belt drive

It is used with shafts arranged with parallel & rotating in opposite direction.



Compound belt drive

→ It is used when power is transmitted from one shaft to another through a no. of pullys.



Velocity Ratio

It is the ratio betn velocity of driver & the driven

Let's consider

d_1 = Diameter of the driver pully.
d_2 = Diameter of the driven pully.
N_1 = speed of driver.
N_2 = speed of driven.

∵ length of belt that passes over driver in one minute

$= \pi d_1 N_1$

Similarly length of the belt that passes over driven pully in one minute $\Rightarrow \pi d_2 N_2$.

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Speed ratio

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1-\frac{s_1}{100}\right)\left(1-\frac{s_2}{100}\right)$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1-\frac{s_1+s_2}{100}\right)$$

$$= \frac{d_1}{d_2}\left(1-\frac{s}{100}\right) \qquad (\because s = s_1+s_2)$$

* If thickness is considered, then velocity ratio

$$\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}\left(1-\frac{s}{100}\right)$$

Creep in belt drive

When the belt passes from slack side to tight side, a certain portion of the belt extends and it contracts again when the belt passes from tight side to slack side. due to change in length, there is a relative motion betn the belt & the pully surfaces, this relative motion is called as creep.

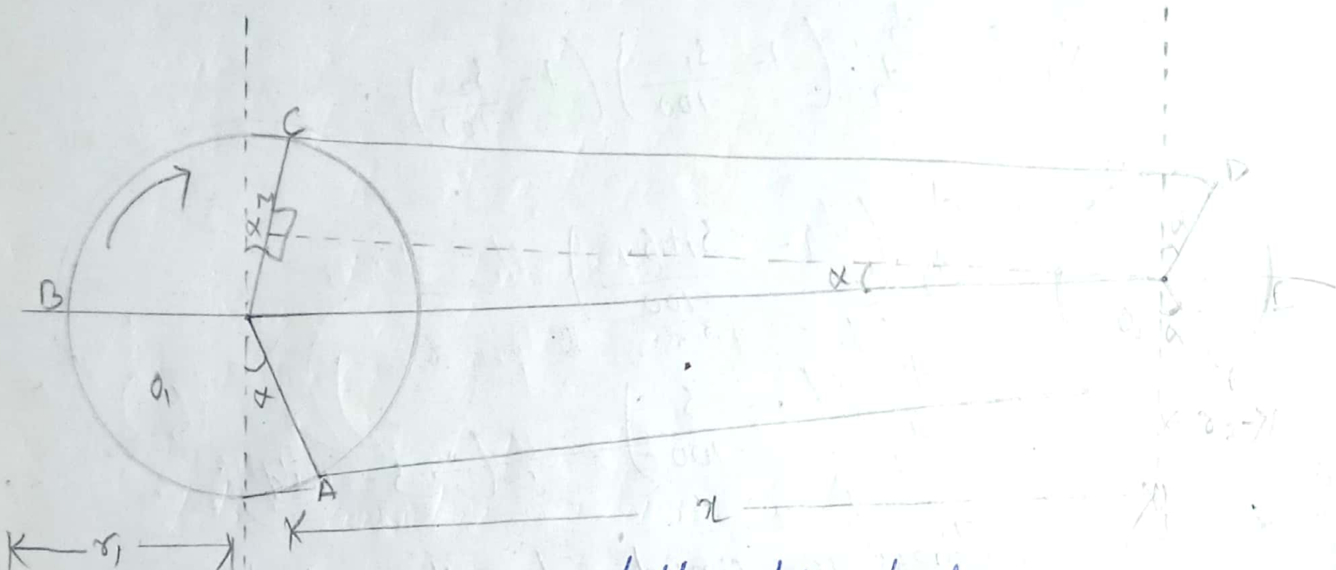
$$\text{speed ratio} = \frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E+\sqrt{\sigma_2}}{E+\sqrt{\sigma_2}}$$

σ_1 = stress in belt on the tight side

σ_2 = stress in belt on the slack side

E = youngs modulus.

Length of open belt drive



Consider a open belt drive having

r_1 = radious of driver pully

r_2 = radious of driven pully

x = distance betn the centre of two pully

L = Total length of the belt.

Let the belt leaves the larger pully at point 'C' & 'A' & the smaller pully at point 'D' & 'F'

Now from 'O_2' draw '$O_2 m$' parallel to 'CD'.

There fore the total length,

$$L = \text{Arc } ABC + CD + \text{Arc } DEF + FA$$

$$L = 2(\text{Arc } BC + CD + \text{Arc } DE) \quad \text{—(1)}$$

$$\text{Arc } BC = (\pi/2 + \alpha) r_1$$

$$\text{Arc } DE = (\pi/2 - \alpha) r_2$$

In $\Delta O_1 O_2 m$

$$\sin\alpha = \frac{O_1 m}{O_2 O_1} = \frac{O_1 C - Cm}{O_1 O_2} = \frac{r_1 - r_2}{x}$$

In ΔO_1O_2m

$$O_2m = \sqrt{(O_1O_2)^2 - (O_1m)^2}$$

$$= \sqrt{x^2 - (r_1-r_2)^2}$$

$$= x\sqrt{1 - \frac{(r_1-r_2)^2}{x^2}}$$

expanding this eqn by binomial theorem

$$O_2m = x - \frac{(r_1-r_2)^2}{2x}$$

Now $O_2m \parallel CD$

$$CD = x - \frac{(r_1-r_2)^2}{2x}$$

Now put the values of arc BC, arc CDE & CD in eqn (i)

$$L = 2\,(\text{Arc } BC + CD + \text{Arc } DE)$$

$$\Rightarrow 2\left[(\pi/2+\alpha)r_1 + x - \frac{(r_1-r_2)^2}{2x} + (\pi/2-\alpha)\, r_2\right]$$

$$= 2\left[r_1\frac{\pi}{2} + r_1\alpha + x - \frac{(r_1-r_2)^2}{2x} + r_2\frac{\pi}{2} - r_2\alpha\right]$$

$$\Rightarrow \left[r_1\pi + 2r_1\alpha + 2x - \frac{(r_1-r_2)^2}{x} + r_2\pi - 2r_2\alpha\right]$$

$$\Rightarrow \left[\pi r_1 + \pi r_2 + 2r_1\alpha - 2r_2\alpha + 2x - \frac{(r_1-r_2)^2}{x}\right]$$

$$\Rightarrow \pi(r_1+r_2) + 2\alpha(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

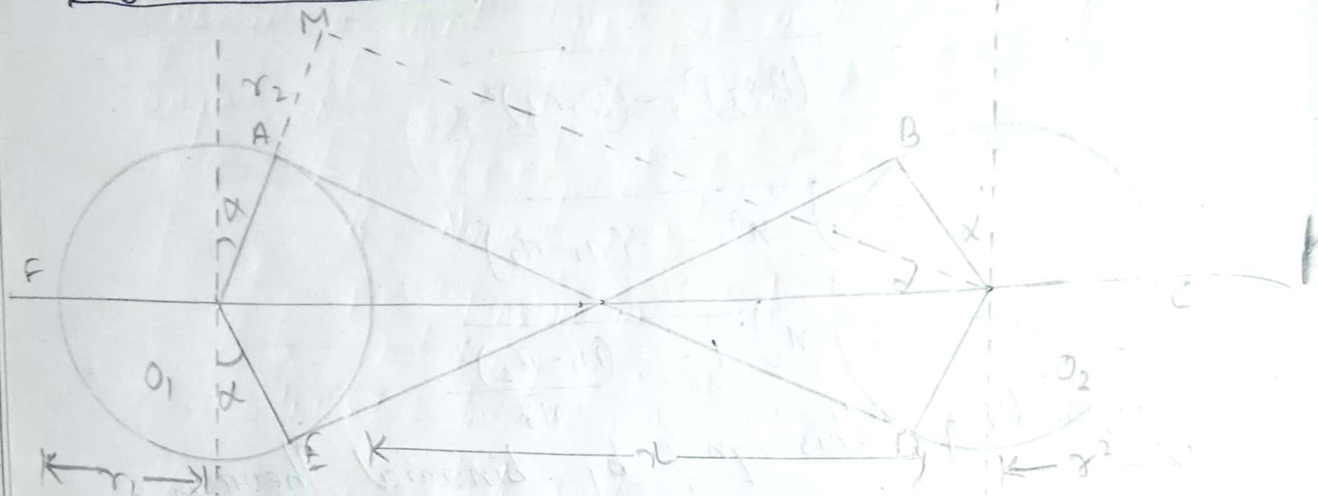
Now putting the value of $\alpha = \frac{r_1-r_2}{x}$

$$L = \pi(r_1+r_2) + 2\left(\frac{r_1-r_2}{x}\right)(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

$$\boxed{L = \pi(r_1+r_2) + 2x + \frac{(r_1+r_2)^2}{x}}$$

↳ Length of open belt drive.

Length of the cross belt drive



Lets consider a cross belt drive having

r_1 = radious of driving pully

r_2 = radious of driven pully

x = Distance betn centre of pully

L = Total length of the belt.

Let the belt lifts larger pully at point A & E & the smaller pulley at point B&D.

Now from O_2, draw O_2M parallel to 'AD'

Total length = Arc EFA + AB + Arc BCD + DE =

2(Arc BE + AB + Arc BD)

Arc AE = $(\pi/2 + \alpha) r_1$

Arc BD = $(\pi/2 + \alpha) r_2$

In ΔO_1O_2M

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1C + AM}{O_1O_2} = \frac{r_1 + r_2}{2}$$

In ΔO_1O_2M

$$O_2M = \sqrt{(O_1O_2)^2 + O_1M^2}$$

$\therefore O_2M = \sqrt{x^2 - (r_1+r_2)^2}$

$$= \sqrt{x^2\left[1 - \frac{(r_1+r_2)^2}{x^2}\right]}$$

$$= x\sqrt{1 - \frac{(r_1+r_2)^2}{x^2}}$$

Expanding this eqn by binomial theorem

$$O_2M = x - \frac{(r_1+r_2)^2}{2x}$$

Now $O_2M \Rightarrow CD = x - \frac{(r_1+r_2)^2}{2x}$

Now, put the value of Arc AE, Arc BD & AB in eqn

$$L = 2\left[Arc\,AE + AB + Arc\,DC\right]$$

$$= 2\,Arc\,AE + 2AB + 2\,Arc\,DC.$$

Ratio of driving tensions for flat belt drive

Consider a driven pully rotating in the clockwise derection. Let

T_1 = Tension in tight side
T_2 = Tension in slack side
θ = Angle of contact
μ = Co-efficient of friction.

Consider a short length of belt subtending an angle of $\delta\theta$ at the centre of the pully

R = Normal reaction

T = Tension in slack side of the element

$T_1 + \delta T$ = Increase in tension on tight side then on slack side

Tension T & $T + \delta T$ at in direction perpendicular to the radii draw at the end of the elements.

The frictional force μR will act tangentially to the pully rim resisting the sliping of the elementary belt on the pully.

Now resolving forces in x direction,

$$R = T\sin\frac{\delta\theta}{2} + (T + \delta T)\sin\frac{\delta\theta}{2}$$

Now put $\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$

$$R = T\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} + \delta T\frac{\delta\theta}{2} \to 0$$

Now neglecting the higher co-efficient

$$R = T\frac{\delta\theta}{2} + T\frac{\delta\theta}{2}$$

$$R = T\left(\frac{\delta\theta}{2} + \frac{\delta\theta}{2}\right)$$

$$\Rightarrow R = T\,\delta\theta \quad \text{——} \ (1)$$

Now resolving forces in y-axis

$$\mu R + T\cos\frac{\delta\theta}{2} = (T+\delta T)\cos\frac{\delta\theta}{2}$$

$$\Rightarrow \mu R + \cancel{T\cos\frac{\delta\theta}{2}} = \cancel{T\cos\frac{\delta\theta}{2}} + \delta T\cos\frac{\delta\theta}{2}$$

Now, Put $\cos\frac{\delta\theta}{2} \Rightarrow 1$

$$\Rightarrow \mu R = \delta T \quad \text{——— (2)}$$

Now putting the value of R in eqn (ii)

$$\mu(T\delta\theta) = \delta T$$

$$\Rightarrow \mu T\delta\theta = \delta T$$

$$\Rightarrow \mu\delta\theta = \frac{\delta T}{T}$$

Now Intigrating both sides

$$\int_0^\theta \mu\,\delta\theta = \int_{T_2}^{T_1}\frac{\delta T}{T}$$

$$\mu\theta = \ln T_1 - \ln T_2$$

$$\mu\theta = \ln(T_1/T_2)$$

$$\boxed{\frac{T_1}{T_2} = e^{\mu\theta}} \text{ — (VVIP)}$$

Power transmitted by belt drive

Let T_1 = Tension in tight side

T_2 = Tension in slack side

Power transmitted by belt drive

$$\boxed{P = (T_1 - T_2)v}$$

where $v = \pi DN/60$ (m/sec)

Centrifugal tension

→ It is defined as the tension occoured by centrifugal force then, this type of tension is called centrifugal tension.

→ It is denoted by T_c.

Maximum tension in the belt

$$\boxed{T_{max} = \sigma_{belt} \times b \times t}$$

where b = width of the belt
t = thickness of the belt
σ_{belt} = stress on the belt.

$\boxed{T_1 = \sigma_{belt} \times b \times t}$ → when $v < 10 m/sec$

$\boxed{T_1 + T_c = \sigma_{belt} + b \times t}$ → when $v > 10 m/sec$.

Angle of contact for open belt drive
$\theta = (180 - 2\alpha) \pi/180$

Angle of contact for cross belt drive
$\theta = (180 + 2\alpha) \pi/180$ ($\pi/180$ = Radian) I.P

i) $T_{max} = 3T_C$

ii) when maximum power occoured, then $v = \sqrt{\frac{T_{max}}{3m}}$

iii) centrifugal tension, $\boxed{T_C = mv^2}$

iv)

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iv) $\rho = \frac{m}{v} \Rightarrow m = \rho v$

$\Rightarrow m = \rho \times l \times b \times t$

Assume leather belt,

$\rho = 1000\ kg/m^3$

$L = 1m$

$\Rightarrow m = 1000 \times 1 \times b \times t$

$\Rightarrow m = 1000\,bt\ kg.$

Initial tension on the belt

→ It is denoted by T_0

→ When a belt is wound around the two pulleys, its two ends are Joint together, so that the belt moves freely over the pulley. Since the motion of the belt is from the driver to driven by the help of a firm grip, inorder to increase the grip, the belt is tightened up. At this stage, the pulleys are stationary the belt is subjected to some tension called initial tension.

$$\boxed{T_0 = \frac{T_1 + T_2}{2}} \rightarrow \text{Neglecting c.f tension}$$

$$\boxed{T_0 = \frac{T_1 + T_2 + 2T_c}{2}} \rightarrow \text{Considering c.f tension}$$

For problem

1) $v = \frac{\pi DN}{60}$ m/sec.

2) $P\ (T_1 - T_2)\ v$

$T_1/T_2 = e^{\mu\theta}$

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Gear

When in a frictional wheel with the teeth cut on it, then it is called as gear.

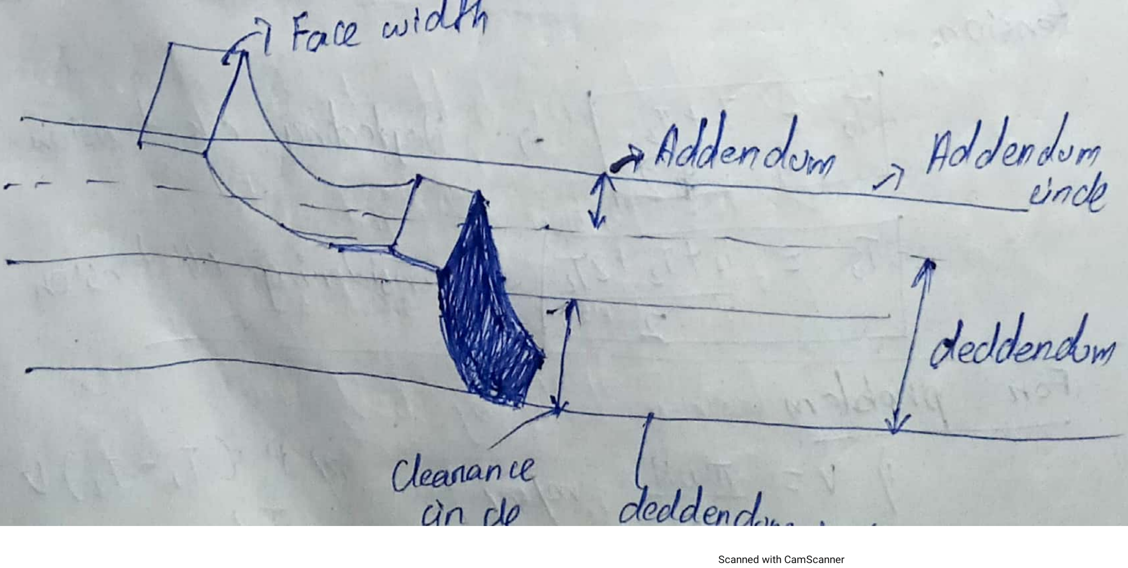
Advantage of Gear drive

- It is used to transmit large power
- It has high efficiency
- It has reliable service
- It has compact layout

Disadvantage of gear drive

- The manufacturing of tool requires special tools & equipments.
- The error in cutting teeth may causes vibration & noise during operation.

Terms used in gear



1) Pitch circle

It is an imaginary circle which is by pure rolling action could give the same motion as actual gear.

2) Addendum circle

It is the circle drawn through the top of the teeth

3) Deddendum circle

It is the circle drawn through the bottom of teeth

4) Addendum

It is the radial distance from addendum circle to pitch circle.

5) Deddendum

It is the radial distance from pitch circle to addendum circle.

6) Circular pitch

→ It is denoted by Pc

→ It is the distance measured on the circumference of the pitch circle from a point of one teeth to corresponding point on the next tooth.

$$\therefore P_c = \frac{\pi D}{T}$$

where D = dia of pitch circle

T = No. of ~~gear~~ teeth on gear

⑦ Diamaterial pitch

→ It is denoted by 'Pd'.

→ It is the ratio betn no. of teeth pitch circular diameter.

$$\boxed{\therefore Pd = \frac{T}{D}}$$

8) Module

→ It is denoted by m

→ It is the ratio betn pitch circular diameter to the no. of teeth.

$$\boxed{\therefore m = D/T}$$

Relation betn 'Pc' & m

We know circular pitch, $P_c = \frac{\pi D}{T}$

$= \pi \times \frac{D}{T} = \pi m$

$$\boxed{\therefore P_c = \pi m}$$

Gear train

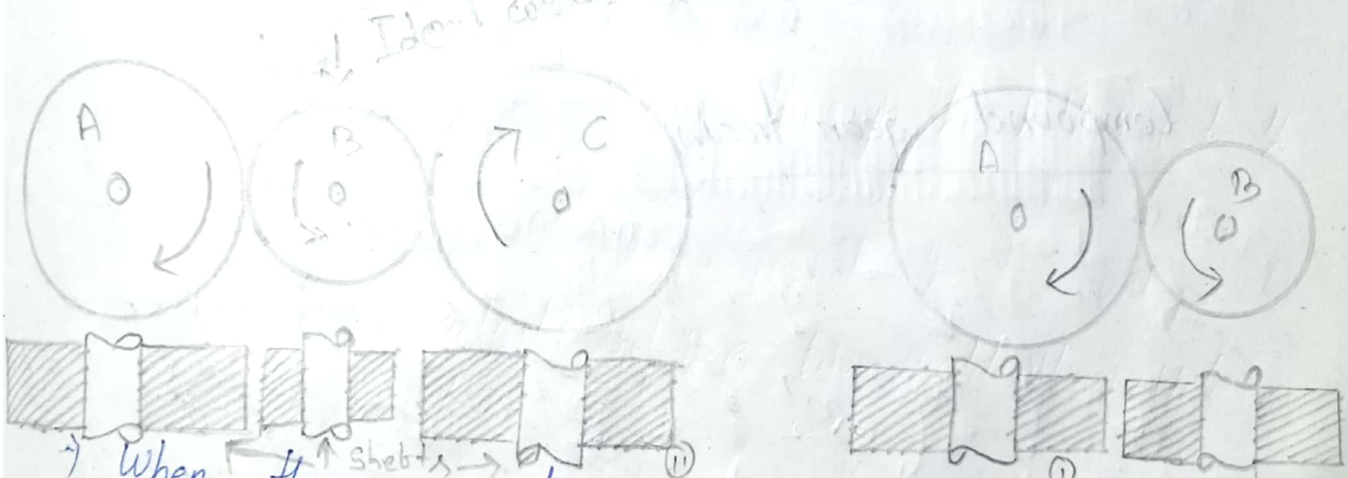
When two or more gears are made to mesh with each other to transmit power from one shaft to another such type of combination is called as gear train.

Classification of gear trains

Gear trains are classified into following types

i) Simple gear trains
ii) Compound gear trains
iii) Reverted gear trains
iv) Epi-cyclic gear train.

Simple gear train



→ When there is only one gear on each shaft, it is known as simple gear train.

→ From fig-(1), the distance betn two shaft is small, the two gears A & B are made to mesh with each other to transmit from one shaft to another.

→ The driven gear rotates in opposite direction of driven gear.

Let N_1 = Speed of gear A
N_2 = Speed of gear B
T_1 = Total gear's of teeth of 'A'
T_2 = total ...

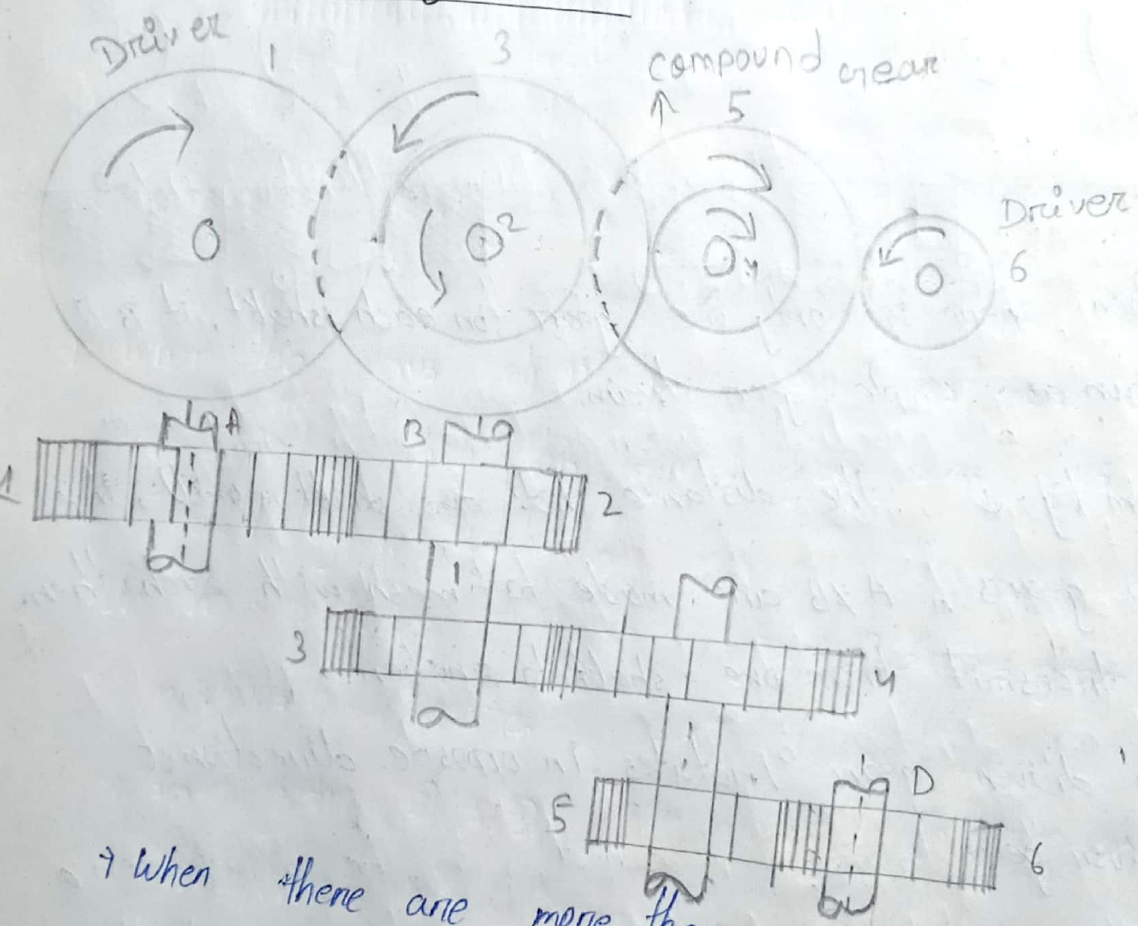
Since, the speed ratio of gear train is the ratio of the speed of the driver to the speed of driven.

$$\therefore \text{Speed ratio} = \boxed{\frac{N_1}{N_2} = \frac{T_2}{T_1}}$$

The train value of gear train is the reciprocal of the speed ratio i.e

$$\text{Train value} = \boxed{\frac{N_2}{N_1} = \frac{T_1}{T_2}}$$

Compound gear train



→ When there are more then one gear on a shaft then this is known as compound gear train.

→ In compound gear train, the gear ① is the driver gear mounded on one shaft gear ② & gear ③ are mounted in a single [illegible]

→ The gear ④ & ⑤ also mounted in a single shaft.

→ Gear ⑥ is the driven gear mounted on a single shaft

Let N_1 = Speed of driver gear ①

T_1 = No. of teeth on driver gear ①

N_2, N_3, N_4, N_5, N_6 are speed of gear 2,3,4,5,6 respectively

T_2, T_3, T_4, T_5, T_6 are no. of teeth of gear 2,3,4,5,6 respectively

we know speed ratio of gear ① & ②

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{— ①}$$

similarly for gear ③ & ④

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \text{— ②}$$

For gear ⑤ & ⑥

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad \text{— ③}$$

Now multiplying eqn ① ② & ③

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

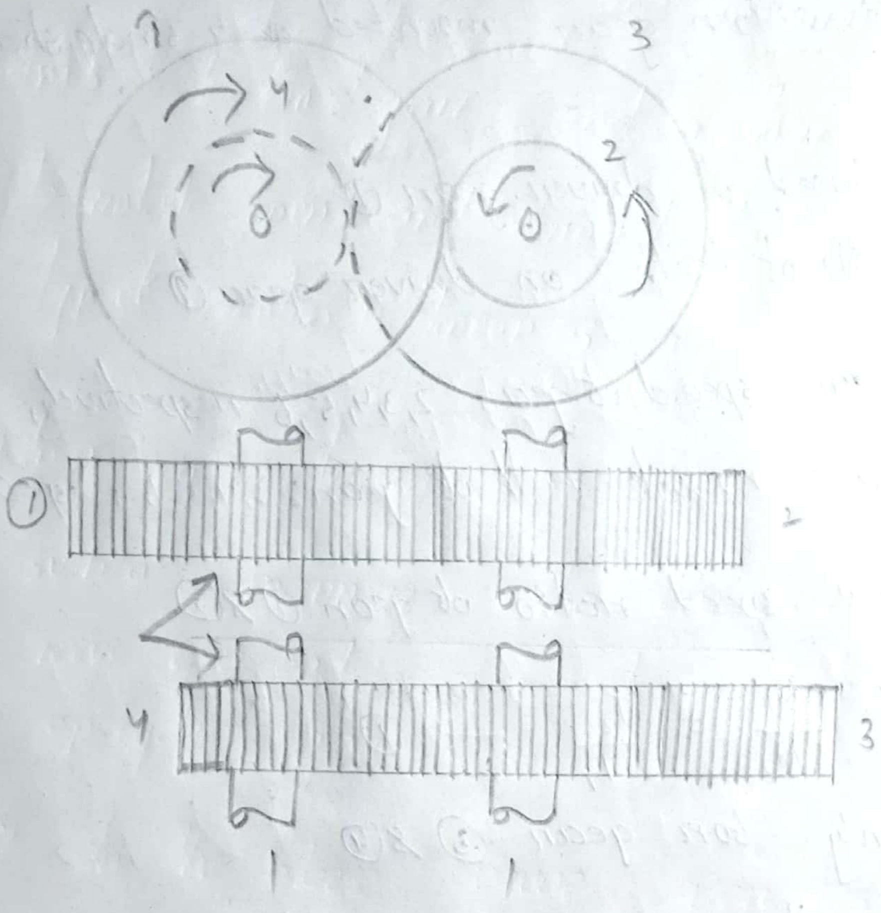
Since gear ② & ③ are mounted on a single shaft

∴ $N_2 = N_3$ & $N_4 = N_5$ (similarly)

so, we can get

$$\boxed{\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}}$$

Reverted Gear train



→ When the axis of 1st gear and last gear are co-axial then this type of gear is called as reverted gear train.

→ In the following figure, gear ① drives gear ② in opposite direction.

→ Since gear ② & ③ are mounted in same shaft, therefore they formed a compound gear & rotates the gear ③ as the same direction of gear ②

→ Gear ③ drives gear ④

so in the reverted gear train the gear ① & ④ is alike.

Let N_1 = Speed of gear ①

T_1 = no. of teeth in gear ①

N_2, N_3, N_4 = Speed of gear 2,3,4 respectively

T_2, T_3, T_4 = No. of teeth on gear 2,3,4 respectively.

r_1 = Pitch circle radious of gear ①

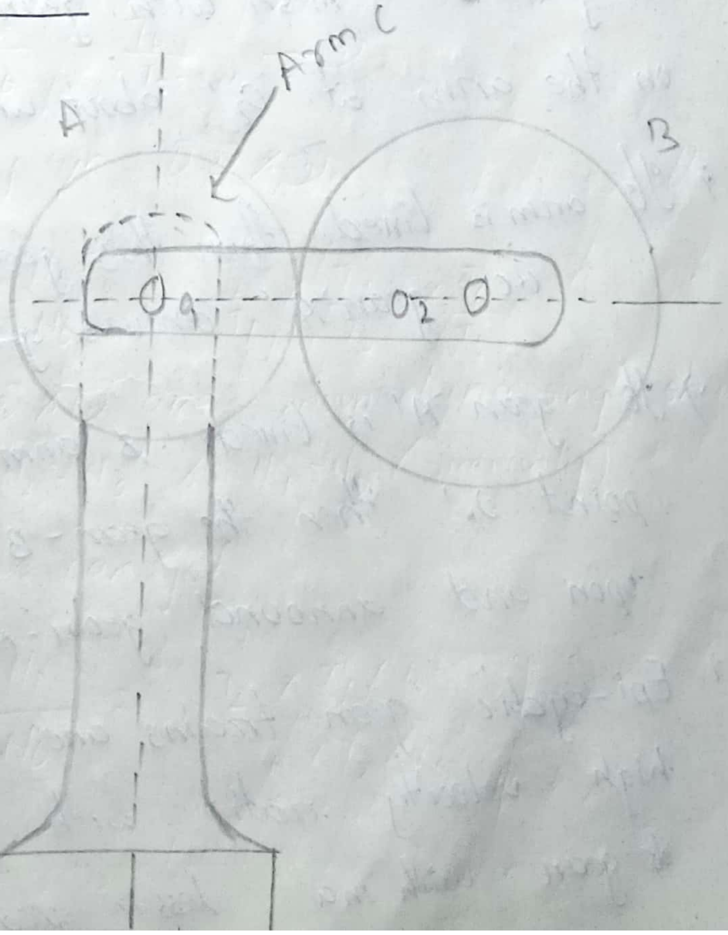
r_2, r_3, r_4 = Pitch circle radious of the gear 2,3,4 respectively

Since the distance betn centres of shaft of gear ① & ② as well as gear ③ & ④ are same

$\therefore r_1 + r_2 = r_3 + r_4$

$\therefore$ Speed ratio = $\frac{N_1}{N_4} = \frac{T_2}{T_1} \times \frac{T_4}{T_3}$

Epicyclic gear train



→ When one gear is fixed and an arm is rotated about the axis of fixed gear and another gear is forced to rotate upon and around the fixed gear then this type of motion is known as epi-cyclic.

→ In a gear train, the gears are arranged in such a manner that one or more of their members move upon and around another member is known as epicyclic gear train.

→ In the above figure, gear 'A' & arm 'C' have a common axis at 'O_1' about which, they can rotate.

→ The gear 'B' mesh with gear 'A' and has it's axis on the arm at 'O_2' about which, gear 'B' can rotate.

→ If arm is fixed then this gear train is simple type i.e. gear 'A' drives gear 'B'.

→ If gear 'A' is fixed & arm is rotated about point 'O_1' then the gear-B is forced to rotate upon and around gear-A.

→ Epi-cyclic gear trains are used for transmitting high velocity ratio with moderate size of gear with in a lesser speed.

Types of Governors

The Governor may broadly be classified into following types
1) Centrifugal governor.
2) Inertia governor.

→ The Centrifugal governor further classified

Centrifugal governors
- Pendulum type
  - Watt governor
- Loaded type
  - Dead weight governor
    - Porter governor
    - Proell governor
  - Spring controlled governor
    - Hartnell governor
    - Hartung governor
    - Wilson-Hartnell governor
    - Pickering governor.

Proell Governor

The proell governor has the ball fixed at B & C to the extension of the link DF, EG. The arm FP & GQ are pivoted at P & Q.

Consider the equilibrium of the forces of one half of the governor. The instantaneous centre (I) lies on the intersection of the line PF produced & the line from D drawn perpendicular to the spindle axis.

Taking moment about I, using the same notations as discussed in fig.

$$F_C \times BM = W \times Im + \frac{W}{2} \times ID = m.g \times Im + \frac{M.g}{2} \times ID_2$$

$$\therefore F_C = m.g \times \frac{Im}{BM} + \frac{M.g}{2}\left(\frac{Im + mD}{BM}\right)$$

$$\dots (\because ID = Im + mD)$$

Multiplying & dividing by Fm, we have

$$F_C = \frac{Fm}{Bm}\left[m.g \times \frac{Im}{Fm} + \frac{M.g}{2}\left(\frac{Im}{Fm} + \frac{mD}{Fm}\right)\right]$$

$$\Rightarrow \frac{Fm}{Bm}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan B)\right]$$

$$\Rightarrow \frac{Fm}{Bm} \times \tan\alpha \left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan B}{\tan\alpha}\right)\right]$$

we know that $F_C = m\omega^2 r$; $\tan\alpha = \frac{r}{h}$ and $q = \frac{\tan B}{\tan\alpha}$

$$\therefore m\omega^2 r = \frac{Fm}{Bm} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1+q)\right]$$

$$\omega^2 = \frac{Fm}{Bm}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{g}{h} \quad -②$$

Substituting $\omega = \frac{2\pi N}{60}$ & $g = 9.81\ m/s^2$, we get

$$N^2 = \frac{Fm}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h} \quad -③$$

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Hartnell governor

A hartnell governor is a springloaded governor. it consists of two bell crank levers pivoted at the point O, O to the frame. The frame is attached to the governor spindle & then rotates with it. A helical spring in compression provides equal downword forces. on the two rollers through the collar on the sleeve.

$m, M \Rightarrow$ Mass of each ball & mass of sleeve in kg

$r_1, r_2 \Rightarrow$ minimum & maximum radious of rotation in meter.

ω_1 & $\omega_2 \Rightarrow$ Angular speed of governor at minimum & maximum radious in rad/sec

S_1 & $S_2 \Rightarrow$ Spring forces exerted at the sleeve at ω_1 & ω_2 in newton

F_{C1} & $F_{C2} \Rightarrow$ Centrifugal force at ω_1 & ω_2

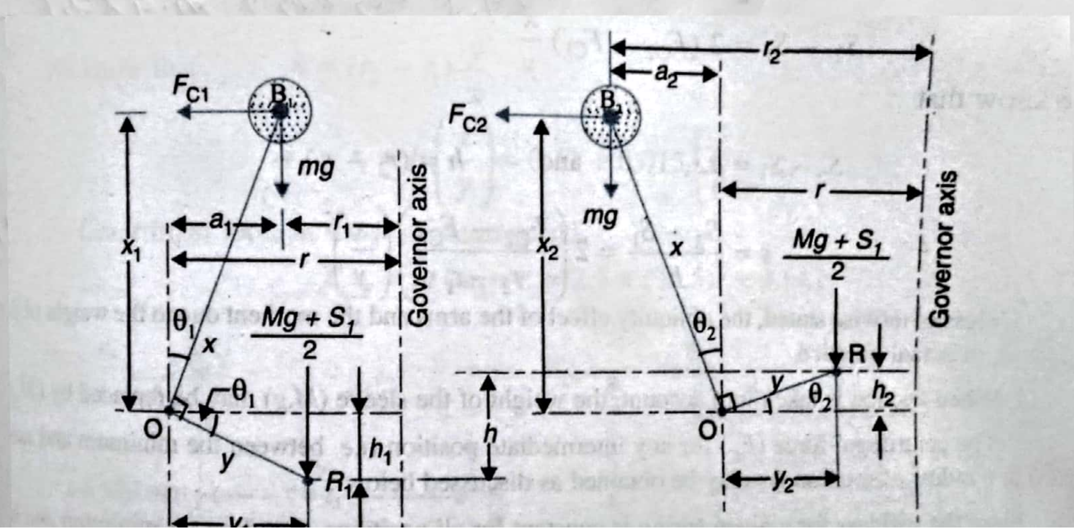
S = Stiffness of the spring

x = length of the vertical on ball arm

y = Length of the horizontal on sleeve arm

r = Distance of fulcrum O from the governer axis.

Consider the forces acting at one bell crak lever.
Let 'h' be the compression of the spring when the radious of rotation changes from $r_1 \rightarrow r_2$.



For the minimum position ie when the radious of rotation changes from r to r_1 as shown in fig

the compression of the spring on the lift of sleeve h_1 is given by $= \frac{h_1}{y} = \frac{a_1}{x} = \frac{r-r_1}{x}$ — ①

Similarly

the compression of the spring on lift of sleeve h_2 is given by $\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x}$ — ②

Adding eqn ① & ②

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad \text{or} \quad \frac{h}{y} = \frac{r_2 - r_1}{x} \quad (\because h = h_1 + h_2)$$

$$h = (r_2 - r_1)\, y/x \quad \text{— ③}$$

Now for minimum position, taking moment about point O we get:

$$\frac{m.g + S_1}{2} \times y_1 = F_{C1} \times x_1 - m.g \times a_1$$

$$m.g + S_1 = \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1) \quad \text{— ④}$$

Again maxm position taking moment about point O, we get

$$\frac{m.g + S_2}{2} \times y_2 = F_{C2} \times x_2 - m.g \times a_2$$

$$m.g + S_2 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) \quad \text{— ⑤}$$

Substracting eqn ④ from eqn ⑤

$$S_2 - S_1 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) - \frac{2}{y_1}(F_{C1} \times x_1 + m.g \times a_1)$$

we know that

$$S_2 - S_1 = h.S \quad \& \quad h = (r_2 - r_1)\, y/x$$

$$\therefore \quad S = \frac{S_2 - S_1}{h} = \left(\frac{S_2 - S_1}{r_2 - r_1}\right) x/y$$

then neglecting the obliquity effect of arm i.e ($x_1 = x_2 = x$ & $y_2 = y_1 = y$). & the moment due to weight of the ball (i.e m.g) we have for minimum position

$$\frac{m \cdot g + S_1}{2} \times y = F_{c_1} \times x \quad \text{or} \quad m \cdot g + S_1 = 2F_{c_1} \times x/y \quad — (6)$$

similarly for maxm position

$$\frac{m \cdot g + S_2}{2} \times y = F_{c_2} \times x \quad \text{or} \quad m \cdot g + S_2 = 2F_{c_2} \times x/y \quad — (7)$$

substracting eqn (7) from (6)

$$S_2 - S_1 = 2(F_{c_2} - F_{c_1}) x/y$$

we know that

$$S_2 - S_1 = h \cdot S \quad \& \quad h = (r_2 - r_1) y/x.$$

$$\therefore \quad S = \frac{S_2 - S_1}{h} = 2\left(\frac{F_{c_2} - F_{c_1}}{r_2 - r_1}\right)\left(x/y\right)^2.$$

Watt Governor

The simplest form of a centrifugal governer is a watt governer. basically a conical pendulum with links attached to a sleeve of negligible mass the arms of governer may be conected to the spindle in following

1) The pivot P, may be on the spindle axis shown in fig
2) The pivot P, may be offset from the spindle axis & the arms when produced intersect at O as shown in fig.
3) The pivot P, may be offset, but the arms cross the axis at O.

h = height of the governer
m = mass of the ball in kg
w = weight of the ball in newton = $m.g$
T = Tension in the arm in newton
ω = Angular velocity of the arm & ball about the spindle axis
r = radious of the path of the rotn of the ball
F_c = Centrifugal force acting on ball

It is assumed that the weight of the arm, links & the sleeve are negligible as compared to the weight of the ball.

Take moments about point O, we have

$$F_c \times h = w \times r = m.g.r$$

$$m.\omega^2.r.h = m.g.r \text{ or } h = g/\omega^2$$

If N is the speed in r.p.m then

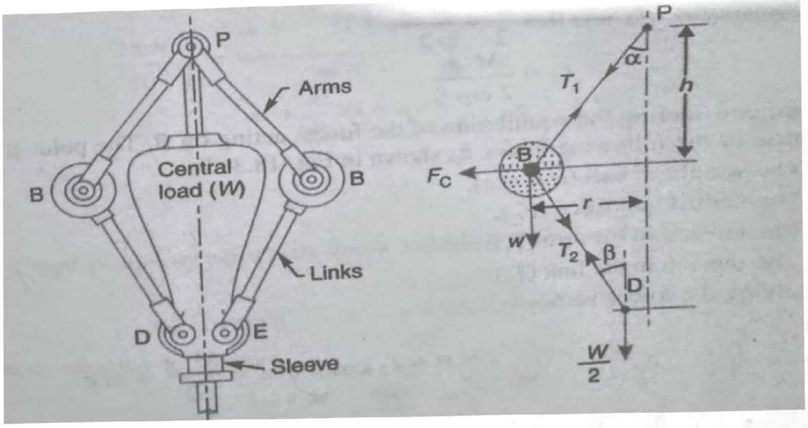
$$\omega = \frac{2\pi N}{60}$$

$$\therefore h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ m.}$$

Met

porter governer

The porter governer is a modification of watt governer.



Let

m = mass of each ball in kg
w = weight of ball in newton.
M = mass of contral load
W = Weight of the central load in newton $= m \cdot g$
r = Radious of rotation in metrem.
h = height of the governor
N = speed on r.p.m
ω = Angular speed.
F_c = Centrifugal force
T_1 = force in the arm
T_2 = Force in the link.
α = Angle of Inclination of the arm
β = Angle of Inclination of the link.

Method of resol^n

Consider the equillibrium of the force acting at 'D'

$$T_2 \cos\beta = \frac{W}{2} = \frac{m \cdot g}{2}$$

$$T_2 = \frac{m \cdot g}{2\cos\beta} \quad \text{——} (1)$$

again consider equillibrium of the force acting at 'B'

Resolving vertically ⇒ $T_2 \cos\alpha = T_2 \cos\beta + w = \frac{m \cdot g}{2} + m \cdot g$

$$\left(\because T_2 \cos\beta = \frac{m g}{2}\right) \quad \text{——} (3)$$

Resolving the forces horizontally.

$$T_1 \sin\alpha + T_2 \sin\beta = F_c$$

$$T_1 \sin\alpha + \frac{m.g}{2\cos\beta} \times \sin\beta = F_c$$

$$T_1 \sin\alpha + \frac{m.g}{2} \times \tan\beta = F_c$$

$$T_1 \sin\alpha = F_c - \frac{m.g}{2} \times \tan\beta \longleftarrow (3)$$

dividing eq^n (3) by eq^n (2)

$$\frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{F_c - \frac{m.g}{2} \times \tan\beta}{\frac{m.g}{2} + m.g}$$

or $\left(\frac{m.g}{2} + m.g\right)\tan\alpha = F_c - \frac{m.g}{2} \times \tan\beta$

$$\frac{m.g}{2} + m.g = \frac{F_c}{\tan\alpha} - \frac{m.g}{2} \times \frac{\tan\beta}{\tan\alpha}$$

substituting $\frac{\tan\beta}{\tan\alpha} = q$ & $\tan\alpha = r/h$, we have

$$\frac{m.g}{2} + m.g = m\,\omega^2 r \times \frac{h}{r} - \frac{m.g}{2} \times q$$

$$m.\omega^2.h = m.g + \frac{m.g}{2}(1+q)$$

$$\therefore h = \left[m.g + \frac{m.g}{2}(1+q)\right]\frac{1}{m.\omega^2} = \frac{m + \frac{m}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

(4)

$$\omega^2 = \left[mg + \frac{mg}{2}(1+q)\right]\frac{1}{m.h} = \frac{m + \frac{m}{2}(1+q)}{m} \times g/h$$

or $$\left(\frac{2\pi N}{60}\right)^2 = \frac{m + \frac{m}{2}(1+q)}{m} \times g/h$$

Scanned with CamScanner

$$\therefore N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}\left(\frac{60}{2\pi}\right)^2$$

$$= \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$

Sensitiveness of governers

N_1 = Minimum equillibrium speed
N_2 = Maximum equillibrium speed
N = Mean equillibrium speed

$$N = \frac{N_1 + N_2}{2}$$

$\therefore$ Sensitivness of governer

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

Stability of governer

A governer is said to be 'stable' when for every speed with in the working range there is a definite configuration. i.e there is only one radious of rotatn of the governer balls at which the governer is in equillibrium.

→ For a stable governer, if the equillibrium speed increases, the radious of governor balls must also increases.

Isochronous governors

+ A governor is said to be Isochronous, when the equilibrium speed is constant (i.e range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The Isochronous is the stage of Infinite sensitivity.

Let us consider the case of porter governor running at speeds N_1 & N_2 r.p.m we have discussed

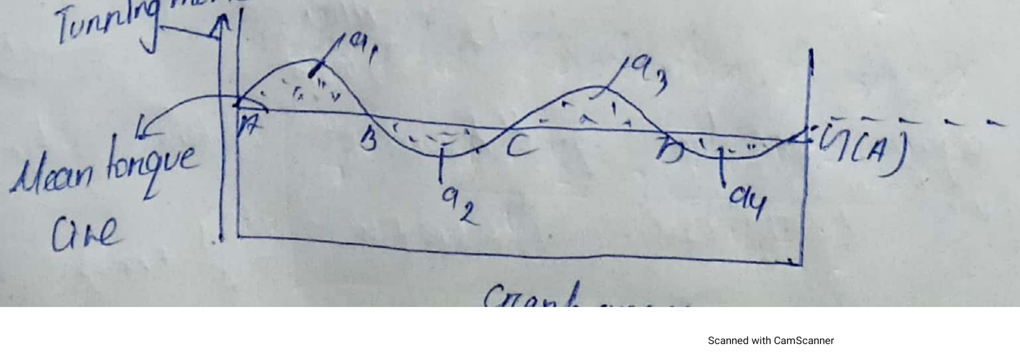
$$(N_1)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_1}$$

$$(N_2)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_2}.$$

Fluctuation of energy

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. The variations of energy above & below the mean resisting torque line are called fluctuation of energy.

Determination of max^m fluctuation of energy



Let the energy in the flywheel at A = E

Energy at B = $E + a_1$

at C = $E + a_1 - a_2$

at D = $E + a_1 - a_2 + a_3$

E = $E + a_1 - a_2 + a_3 - a_4$

F = $E + a_1 - a_2 + a_3 - a_4 + a_5$

G = $E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

Let us now suppose that the greatest of energies is at B and least at E

∴ max^m energy in fly wheel

$= E + a_1$

min^m energy in fly wheel

$= E + a_1 - a_2 + a_3 - a_4$

∴ max^m fluctuation of energy

ΔE = max^m energy - min^m energy

$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$

Coefficient of Fluctuation of energy

It may be defined as the ratio of the max^m fluctuation of energy to the W.D per cycle.

m.m = $C_E = \dfrac{\text{max}^m \text{ fluctuation of energy}}{\text{W.D / cycle}}$

Flywheel

A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, & release it during the period when the requirement of energy is more then the supply.

or

A flywheel controles the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.

Coefficient of fluctuation of speed

The difference betn the maxm & minimum speeds during a cycle is called the maxm fluctuation of speed.

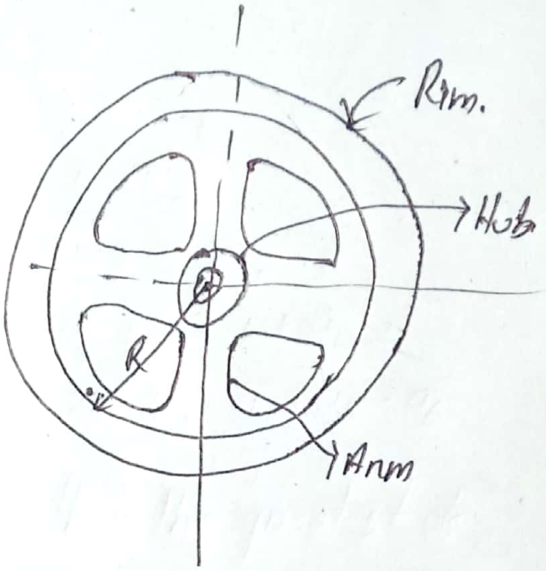
N_1 & N_2 => maxm & minm speed in r.p.m

N -> mean speed $= \frac{N_1+N_2}{2}$

∴ Coefficient of fluctuation of speed

$$\boxed{C_s = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}}$$

Energy stored in flywheel



Let m = mass of flywheel

k = Radius of gyration of the flywheel in meters

I = Mass moment of inertia of the flywheel

N_1 & N_2 → max^m & min^m speed.

ω_1 & ω_2 ⇒ max^m & min^m angular speed.

we know mean KE of the flywheel

$$E = \frac{1}{2} \times I \cdot \omega^2 = \frac{1}{2} \times m \cdot k^2 \cdot \omega^2$$

then max^m fluctuation of energy

$$\Delta E = \text{Max}^m\, KE - \text{min}^m\, KE$$

$$\Rightarrow \frac{1}{2} \times I(\omega_1)^2 - \frac{1}{2} \times I(\omega_2)^2 = \frac{1}{2} \times I\left[(\omega_1)^2 - (\omega_2)^2\right]$$

$$\Rightarrow \frac{1}{2} \times I\,(\omega_1 + \omega_2)(\omega_1 - \omega_2) = I\omega(\omega_1 - \omega_2) \quad \text{— (1)}$$

$$= I\omega^2\left(\frac{\omega_1 - \omega_2}{\omega}\right)$$

$$\Rightarrow I \cdot \omega^2 \cdot C_S = mk^2 \cdot \omega^2 \cdot C_S \quad \text{— (2)}$$

$$= 2 \cdot E \cdot C_S \quad \text{— (3)}$$

substract $k = R$ in eq (2)

$$\Delta E = m \cdot R^2 \times \omega^2 \cdot C_S = m \cdot v^2 \cdot C_S.$$

Static Balancing

→ A system of rotating masses is said to be in static balance if the combined mass centre of the system lies on the axis of rotation.

→ For static balancing, the vector sum of all the force acting on the rotor (rotating body) is zero.

i.e- $m_1 r_1 \omega^2 + m_2 r_2 \omega^2 + m_3 r_3 \omega^2 + \ldots = 0$.

Where, m = mass (kg.)

r = radius of rotation

ω = Angular velocity, rad/sec.

Q Explain the causes and effects of unbalance? (E.O)

Ans

Causes of unbalance

→ Faulty design or Manufacture of shafts or rotors. (bent shafts) etc.

→ Non homogenity of materials.

→ Faulty mounting of parts causes eccentricity.

→ Misalignment of bearings.

→ plastic deformation of certain parts.

→ Weak foundation, loose fittings etc.

→ Thermal gradient, cavitation hammering etc.

→ Non symmetry of the parts.

→ Unbalanced centrifugal force in the system.

→ External Excitation applied on the system.

Effects of unbalance

→ Vibration in the rotating machinery is the main effect of unbalance.

→ Uneven wear and tear of rotating parts.

→ Quick damage of bearing.

→ Premature failure of the rotating parts.

→ Abnormal Sound (noise), high friction etc. in the rotating parts.

→ Decreased life of bearings.

→ Reduced machine life.

Q What is the necessity of balancing of rotating mass?

Ans
→ Rotating masses on high speed engines or machines need to be balanced as far as possible inorder to avoid dynamic forces to be imparted on them which will cause increase in the loads in bearings and various stresses on their members.

→ It is necessary for avoiding unpleasant and even dangerous vibration.

Q Difference between static & dynamic balancing? (E.O)

Ans

Static balancing	Dynamic Balancing.
→ Static balancing would refers to balancing, in a single plane.	→ Dynamic balancing would refers to balancing, in more than one plane.
→ It is also known as primary balancing.	→ It is also known as secondary balancing.
→ It is a balance of forces due to action of gravity	→ It is a balance due to action of inertia forces.
→ Rotation of flywheels, grinding wheels, car wheels are treated as static balancing problems.	→ Rotation of shaft of turbo-generator is a case of dynamic balancing problems.
→ It occurs when the centre of gravity of an object is on the axis of rotation.	→ It occurs when the rotation does not produce any resultant centrifugal force or couple. Here the mass of axis is coincidental with the rotational axis.

Vibration of Machine parts

Amplitude

The maximum displacement of a vibrating body from its equilibrium position is known as amplitude.

Time period

It is the time taken by a motion to repeat itself and is measured in seconds.

Frequency

→ It is the no. of cycles of motion completed in one second.

→ It is expressed in hertz (Hz) and is equal to one cycle per second.

Vibration

When an elastic body (shaft, bearings etc.) which is fixed at one end and is displaced at other end from its equilibrium position by the application of external forces, the body ~~move~~ starts to move to and fro. Then the body is said to be in vibration.

Types of Vibrations

(i) Free / Natural Vibration

→ When no external force acts on the body, after giving it an initial displacement, then the body is said to be under free vibration.

→ The frequency of free vibration is called free or, natural frequency.

(ii) Forced Vibration

→ When a repeated force continuously acts on a system, the vibrations are said to be forced.

→ The frequency of the vibrations is that type of applied force and is independent of their natural frequency of vibrations.

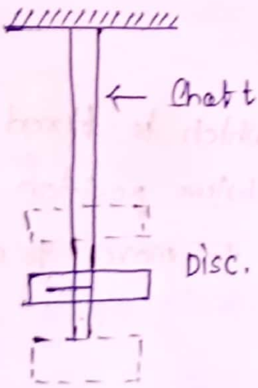
(iii) Damped Vibration

→ When there is a reduction in amplitude over every cycle of vibrations, the motion is said to be damped vibration.

Types of free Vibrations (E·O)

Consider a vibrating body, e.g - rod, shaft or spring. Figure shows a mass less shaft, one end of which is fixed and the other end carrying a heavy disc. The system can execute the following types of vibration.

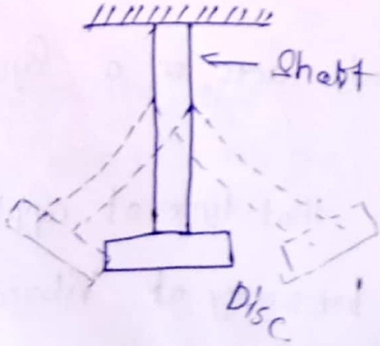
(i) Longitudinal Vibrations



→ If the shaft is elongated and shortened so that the same moves up and down resulting in tensile and compressive stresses in the shaft, the vibrations are said to be longitudinal.

→ The different particles of the body move parallel to the axis of the body.

(ii) Transverse Vibrations



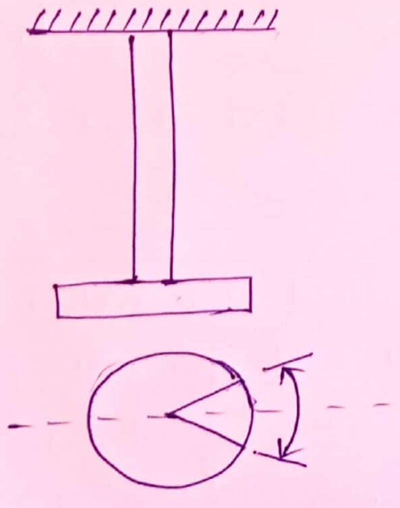
→ When the shaft is bent alternately, and tensile & compressive stresses due to bending result, the vibrations are said to be transverse.

→ The particles of the body move approximately perpendicular to its axis

(iii) Torsional Vibrations

→ When the shaft is twisted and untwisted ~~and untwisted~~ alternately and torsional shear stresses are induced, the vibrations are known as torsional vibrations.

→ The particles of the body move in a circle about the axis of the shaft



Causes of Vibrations (E.O)

→ Unbalanced reciprocating machine parts.

→ Unbalanced rotating machine parts.

→ Incorrect allignment of the transmission elements such as coupling etc.

→ Use of simple spur gears for power transmission.

→ Worn-out teeth of the gears for power transmission.

→ Loose transmission of belts & chains.

→ Loose fastenings of the moving parts.

Remedies of Vibration

Although it is impossible to eliminate the vibrations, yet these can be reduced by adopting various remedies. Some of the remedies are listed below

c) → Partial balancing of reciprocating masses.

→ Balancing of unbalanced rotating masses.

→ Using helical gears instead of spur gears.

→ Proper tightening & locking of fastening and periodically ensuring it q

→ Correcting the misalignment of rotating components and checking it from time to time.

→ ~~Timp~~ Timely replacement of workout moving parts, slides & bearings with excessive clearance.